Mark Scheme (Results)

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Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Further Pure Mathematics Marking
(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:
1. <u>Factorisation</u>:

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x =$….

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x =$….

2. <u>Formula</u>:

Attempt to use the **correct** formula (shown explicitly or implied by working)

with values for a, b and c, leading ccto $x =$….

3. <u>Completing the square</u>:

$x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x =$….

Method marks for differentiation and integration:
1. <u>Differentiation</u>

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. <u>Integration</u>:

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula:
Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is <u>not</u> quoted, the method mark can be gained by implication from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:
The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is give

Question	Scheme	Marks
1	$$[\pm(2k+6)]^2 - 4\times k\times 16$$ $$4k^2 - 40k + 36 = 0$$ $$\rightarrow \{4\}(k-1)(k-9)\{=0\} \rightarrow k = ...$$ $$k = 1, 9$$	M1 dM1 M1 A1 [4]
		Total 4 marks

Mark	Notes
M1	**Allow working on expression, equation or any inequality for all M marks** Applies $b^2 - 4ac = [\pm(2k+6)]^2 - 4\times k\times 16$ and might be seen embedded in an attempt at the quadratic formula.
dM1	Forms a 3TQ in k by attempting to expand and collect the like terms.
M1	Attempts to solve their 3TQ in k to find at least one real value of k by any correct method (See General Principles). Condone labelling their $\boldsymbol{k}$ as $\boldsymbol{x}$ for this method mark. If there is no method shown, (use of a calculator) then the 3TQ must be correct and both k values must be fully correct ($k = 1$, $k = 9$) for evidence for this mark.
A1	For $k = 1, 9$ selected as their final answers. (Extra inequality answers for k must be rejected)

Question	Scheme	Marks
1 **ALT I** **(Via completing Square)**	$$k(x - \frac{k+3}{k})^2 - \frac{(k+3)^2}{k} + 16$$ $$16 - \frac{(k+3)^2}{k} = 0 \rightarrow k^2 - 10k + 9 = 0$$ $$(k-1)(k-9)\{=0\} \rightarrow k = ...$$ $$k = 1, 9$$	M1 dM1 M1 A1 [4]
		Total 4 marks

Mark	Notes
M1	Attempts to complete the square for $kx^2 - (2k+6)x + 16$ (See General Principles) Similarly, divides both sides of the given quadratic equation by k first, then attempts to complete the square: $(x - \frac{k+3}{k})^2 - \frac{(k+3)^2}{k^2} + \frac{16}{k}$
dM1	Sets the y coordinate of their turning point to be zero ($= 0$ can be implied) attempts to expand and collect like terms.
M1	Solves their 3TQ or polynomial to find at least one real value of k by any correct method (See General Principles) If there is no method shown, (use of a calculator) then their 3TQ must be correct and both k values must be fully correct ($k =1$, $k = 9$)for evidence for this mark.
A1	For $k = 1, 9$ only

Question	Scheme	Marks
1 **ALT II** **(Via** **sum and** **product** **of roots)**	$\alpha + \beta = \pm\dfrac{2k+6}{k}, \alpha \times \beta = \dfrac{16}{k} \rightarrow \left\{ \alpha + \alpha = \pm\dfrac{2k+6}{k}, \alpha \times \alpha = \dfrac{16}{k} \right\}$	M1
	$(\dfrac{k+3}{k})^2 = \dfrac{16}{k} \rightarrow k^2 - 10k + 9 = 0$	dM1
	$(k-1)(k-9)\{=0\} \rightarrow k = ...$	M1
	$k = 1 , 9$	A1
		[4]
		Total 4 marks

Mark	Notes
M1	For: $\alpha + \beta = \pm\dfrac{2k+6}{k}, \alpha \times \beta = \dfrac{16}{k} \rightarrow \left\{ \alpha + \alpha = \pm\dfrac{2k+6}{k}, \alpha \times \alpha = \dfrac{16}{k} \right\}$
dM1	Uses the fact that the quadratic equation has equal roots $\alpha = \beta,$ sets up a correct equation in k, expands, collects like terms to form a 3TQ (or a Cubic equation) e.g. $\left(\dfrac{2k+6}{2k}\right)^2 = \dfrac{16}{k} \rightarrow 4k^3 - 40k^2 + 36k = 0$ or $4k^2 - 40k + 36 = 0$
M1	Solves their 3TQ or Cubic equation to find at least one real value of k by any correct method (See General Principles) If there is no method shown, (use of a calculator) then their 3TQ or Cubic must be correct and both k values must be fully correct ($k = 1, k = 9$)for evidence for this mark.
A1	For $k = 1, 9$ only

ALT III: (**Via differentiation**)

M1: Differentiates $kx^2 - (2k+6)x + 16$ with respect to x to achieve a linear expression

dM1: Sets their derivative to 0, solves for x and substitutes x back to $kx^2 - (2k+6)x + 16 = 0$ to form a 3TQ.
M1A1 same as ALT II

e.g. $kx^2 - (2k+6)x + 16 \rightarrow 2kx - 2k - 6 = 0 \rightarrow x = 1 + \dfrac{3}{k}$

$\rightarrow k\left(1 + \dfrac{3}{k}\right)^2 - (2k+6)(1 + \dfrac{3}{k}) + 16 = 0$

$\rightarrow k = ...$

Send to review if not sure

Question	Scheme	Marks
2(a)	$$\frac{2}{\sqrt{1+3x}} = 2(1+3x)^{\left(-\frac{1}{2}\right)}$$	B1
	$$\{2\}\left(1+\left(-\frac{1}{2}\right)(3x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(3x)^2}{2!}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(3x)^3}{3!}+...\right)$$	M1A1
	$$2-3x+\frac{27x^2}{4}-\frac{135x^3}{8}+...$$	A1 [4]
(b)	$$-\frac{1}{3}<x<\frac{1}{3} \qquad \text{(Accept } |x|<\frac{1}{3}\text{)}$$	B1 [1]
		Total 5 marks

Part	Mark	Notes
(a)	B1	Correct simplification of the given expression. This mark may be implied by a correct expansion.
	M1	Attempts at the binomial expansion $(1+3x)^{\pm\frac{1}{2}}$ or $2(1+3x)^{\pm\frac{1}{2}}$ For an attempt at the binomial expansion. • The first term is 1 or 2 • The powers of $3x$ are correct in all terms, e.g. $(3x)^2$ • The correct denominators are used, 2! and 3! oe
	A1	Allow this mark for at least 1 correct algebraic term, correctly simplified, from $$-3x+\frac{27x^2}{4}-\frac{135x^3}{8} \text{ or } -\frac{3}{2}x+\frac{27x^2}{8}-\frac{135x^3}{16}$$
	A1	Correct simplified expansion in ascending order, coefficients must be in simplest **fractions**. (Ignore extra terms with powers > 3) Do not isw
(b)	B1	For the correct range of values of x. $-\frac{1}{3}<x<\frac{1}{3}$ or $|x|<\frac{1}{3}$, do not accept $|3x|<1$

ALT for (a) Uses **Maclaurin's expansion (If seen send to review)**

$$f(x) = f(0) + f'(0)x + \frac{f''(x)}{2!}x^2 + \frac{f'''(x)}{3!}x^3+...$$

B1 Correct simplification of the given expression.

M1 Achieves $f'(x) = P(1+3x)^{-\frac{3}{2}}, f''(x) = Q(1+3x)^{-\frac{5}{2}}, f'''(x) = R(1+3x)^{-\frac{7}{2}}, P, Q, R \neq 0$ and attempts to find the values of $f'(0), f''(0)$ and $f'''(0)$

A1 Correct unsimplified expansion OR at least 2 correct simplified terms

A1 Fully correct simplified expansion. (Ignore extra terms with powers higher than 3)

For reference, the correct derivatives are:

$$f'(x) = -3(1+3x)^{-\frac{3}{2}}, f''(x) = \frac{27}{2}(1+3x)^{-\frac{5}{2}}, f'''(x) = \frac{-405}{4}(1+3x)^{-\frac{7}{2}}$$

Question	Scheme	Marks
3(a)	$$\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB} = 7i + 2aj - (i + 3aj) = 6i - aj$$	M1A1
	$$\left|\overrightarrow{OA}\right| = \sqrt{6^2 + a^2} = 3\sqrt{5} \Rightarrow 6^2 + a^2 = 45 \Rightarrow a^2 = 9 \Rightarrow a = 3$$	M1A1 [4]
(b)	$$\overrightarrow{OA} = 6i - '3'\, j$$	B1ft
	{Unit vector parallel to $\overrightarrow{OA}$ is} $\{\pm\}\dfrac{1}{3\sqrt{5}}(6i - 3j)$ oe ,isw	B1 [2]
		Total 6 marks

Part	Mark	Notes
(a)	M1	For the correct vector statement with vector $\overrightarrow{OA}$ or $\overrightarrow{AO}$ e.g. $\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB}$ or $\{\overrightarrow{OA} =\} 7i + 2aj - (i + 3aj)$ oe
	A1	For the correct simplified or unsimplified vector $\overrightarrow{OA}$ **or** $\overrightarrow{AO}$
	M1	For using the correct Pythagoras theorem on their $\overrightarrow{OA}$ or $\overrightarrow{AO}$ with $3\sqrt{5}$, leading to a value for a^2 or a
	A1	For the correct value of $a=3$ only (must reject $a=-3$ if present) **NB:A value of $a=3$ must come from a correct vector of OA with no incorrect work seen.**
(b)	B1ft	For the correct vector for $\overrightarrow{OA}$ or $\overrightarrow{AO}$ using their a (which $a>0$), (Substitutes their a value to $\overrightarrow{OA} = 6i - aj$ or $\overrightarrow{AO} = -6i + aj$
	B1	Correct unit vector $\dfrac{1}{3\sqrt{5}}(6i - 3j)$ oe, can be in either direction, so accept $+$ or $-$ isw

Extra notes:

1. Allow working in column vectors throughout.

2. Condone missing arrow on their vectors.

3. **Special Case for part a:** Correct answer with minimum work, no incorrect working shown
e.g. $\sqrt{6^2 + a^2} = 3\sqrt{5}$ M1A1M1 $\rightarrow 6^2 + a^2 = 45 \rightarrow a^2 = 9 \rightarrow a = 3$ A1

Question	Scheme	Marks
4(a)	$$f'(x) = 3px^2 + 2qx - 37$$ $$f'(-2) = 3p(-2)^2 + 2q(-2) - 37 = -33$$ $$f(-5) = p(-5)^3 + q(-5)^2 - 37(-5) - 12q = 0$$ $$\begin{cases} 12p - 4q = 4 \\ -125p + 13q = -185 \end{cases} \Rightarrow p = \cdots \ q = \cdots$$ $$p = 2, q = 5$$	M1 M1 M1 dM1 A1A1 [6]
(b)	$$\begin{array}{r} 2x^2 - "5"\,x - "12" \\ x+5 \overline{)\,2x^3 + "5"\,x^2 - 37x - "60"} \end{array}$$ $$OR$$ $$2x^3 + "5"x^2 - 37x - "60" = (x+5)(Ax^2 + Bx + C)$$ $$\Rightarrow A = 2, B = \cdots, C = \cdots$$ $$2x^2 - 5x - 12 = (2x+3)(x-4)$$ $$\{f(x) = \}(x+5)(2x+3)(x-4)$$	M1 M1 A1 [3]
(c)	$$(x+5)(2x+3)(x-4) = 0, \Rightarrow x = -5, \ 4, \ -\frac{3}{2}$$	M1A1 [2]
		Total 11 marks

Part	Mark	Notes
(a) Mark i and ii together	M1	For differentiating the given expression in terms of p and q with respect to x, at least one term correct and no terms integrated.
	M1	Attempts $f'(\pm 2) = -33$
	M1	Attempts $f(\pm 5) = 0$
	dM1	For solving the two resulting simultaneous equations by any correct method (allow slips) providing there are two variables p and q in both, leading a value of p or q. Explicit method to solve must been seen. **Depends on all previous method marks**
	A1	For $p = 2$ or $q = 5$ from correct working in two correct equations
	A1	For both $p = 2$ and $q = 5$ from correct working
(b)	M1	For using polynomial division (or any other **algebraic** method) to find the quadratic factor. **Division:** Look for divisor of $(x+5)$, quotient of a quadratic $2x^2 -$ their $qx + c$ $(c \neq 0)$ **Compare coefficients**: minimum required is an identity of the correct form followed by values of $A = 2, B = -$their $q, C = \ldots (C \neq 0)$
	M1	Correct method of factorising their quadratic factor. (See General Principles)
	A1	Correct factorised expression written in full on one line.
(c)	M1	Correct method of solving their $f(x) = 0$. Their $f(x)$ must have 3 solutions, one of which must be $x = -5$, "=0" or their method may be implied by their x values. They must use their result in part b which $f(x)$ reaches 2 or 3 factors, if part b is not attempted, a full algebraic method of solving $f(x)=0$ must be seen in this part.
	A1	For all three correct values of x, no extra solutions.

(a) **ALT I Algebraic Division**

M1 Same as main scheme
M1 Their f'(x) is in the form of a 3TQ, look for divisor of (x+2), quotient of a linear expression leading to a constant remainder in **p** and **q**, sets = -33
M1 For dividing f(x), look for divisor of (x+5), quotient of a 3TQ, leading to a constant remainder in **p** and **q,** sets=0
dM1A1 Same as main scheme

You may also see variations for their division, marking is similar as above, (if not sure, send to review) e.g.
For f'(x)

$$
\begin{array}{r|ccc}
-2 & 3p & 2q & -37 \\
& & -6p & 12p-4q \\
\hline
& 3p & -7p+2q & 12p-4q-37=-33
\end{array}
$$

For f(x)

$$
\begin{array}{r|cccc}
-5 & p & q & -37 & -12q \\
& & -5p & -5q+25p & 25q-125p+185 \\
\hline
& p & q-5p & -5+25p-37 & 13q-125p+185=0
\end{array}
$$

(a) **ALT II Compare coefficients**
M1 Same as main scheme
M1 Sets their f'(x) which is in the form of a 3TQ, equal to (x+2)(Kx+C)-33 (K≠ 0, $C \neq 0$) , then attempts to equate the coefficients of x to form an equation in **p** and **q**
M1 Sets f(x)=(x+5) $[px^2 + Rx + S]$ ($R \neq 0, S \neq 0$), then attempts to equate their coefficients of x to form an equation in **p** and **q**.
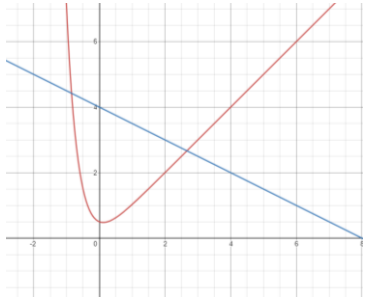**dM1A1** Same as main scheme

For reference, the correct working for this method:
$$f'(x) = 3px^2 + 2qx - 37$$
$$= (x + 2)(3px - 2) - 33$$
$$\Rightarrow 6px - 2x = 2qx$$
$$6p - 2 = 2q$$
$$f(x) = px^3 + qx^2 - 37x - 12q = (x + 5)\left[px^2 + (q - 5p)x - \frac{12q}{5}\right]$$
$$\Rightarrow 5(q - 5p)x - \frac{12q}{5}x = -37x$$
$$5(q - 5p) - \frac{12q}{5} = -37$$
$$\frac{13q}{5} - 25p = -37$$

Question	Scheme	Marks
5	$$\frac{dr}{dt} = 0.7$$	B1
	$$\{\frac{dF}{dr}\} = -2 \times \frac{3}{20} r^{-3} \text{ oe}$$	M1A1
	$$\{\frac{dF}{dt} = \frac{dF}{dr} \times \frac{dr}{dt} = \} - \frac{0.3}{2.8^3} \times 0.7 = -0.0095663 \dots \text{ (newtons/s)}$$	M1dM1A1 [6]
	Accept **awrt** -0.00957 or -9.57×10^{-3}	
		Total 6 marks

Mark	Notes
B1	For writing $\frac{dr}{dt} = 0.7$ can be implied or embedded in later work. Condone using letter **d** or **D** or **R** only instead of **r.**
M1	For an attempt to differentiate the formula for *F,* for $\dots r^{-2} \to \dots r^{-3}$ oe Attempts quotient rule, look for $\frac{-Pr}{Qr^4}$ (*P > 0, Q > 0*) oe, if the quotient rule is quoted, it must be correct, if clearly they use incorrect quotient rule M0
A1	For a correct differentiated expression simplified or unsimplified
M1	For a **correct** chain rule, accept in any order. (correct chain rule stated or implied by their working) Condone using letter d instead of r. For example, accept $\frac{dF}{dr} = \frac{dF}{dt} \times \frac{dt}{dr}$ *or* $\frac{dF}{dr} = \frac{dF}{dt} \div \frac{dr}{dt}$ $\left(\frac{dF}{dt} \text{ does not need to be the subject} \right)$
dM1	For substituting the value of *r=2.8* into their dF/dr in a **correct** chain rule, leading to $\frac{dF}{dt} = \dots$ (a value, not a numerical expression) , the value can be a fraction or decimal. If their final answer is not correct, explicit substitution must be seen. **Depends on the previous method mark only.**
A1	For the correct value of $\frac{dF}{dt}$ (Condone lack of units or incorrect units) isw

Question	Scheme	Marks
6	$\log_2(8-3x) - 1 = -4x - 1 \Rightarrow \log_2(8-3x) - \log_2 2 = -4x - 1$	M1
	$\Rightarrow \log_2\left(\dfrac{8-3x}{2}\right) = -4x - 1$	
	$\Rightarrow \dfrac{8-3x}{2} = 2^{-4x-1}$	M1
	$\Rightarrow 4 - \dfrac{3}{2}x + x = x + 2^{-(4x+1)}$	M1
	$\Rightarrow 4 - \dfrac{1}{2}x = x + 2^{-(4x+1)}$	M1A1
	Draws the line $y = 4 - \dfrac{x}{2}$ For the values $x = -0.8/0.9$ and $x = 2.6/2.7$	M1 A1 [7]
		Total 7 marks

Mark	Notes
	General principles of marking this question for the first 5 marks **Award the first two marks when you see them.**
M1	Award the first M mark for dealing with the log wherever you see it For example: $\log_2(8-3x) = -4x \Rightarrow 8 - 3x = 2^{-4x}$
M1	Award the second M mark for either adding -1 to **both** sides and converting $-1 = \log_2 2$, **or** by dividing throughout by 2 and dealing with the power of -1. For example: $\dfrac{8-3x}{2} = \dfrac{2^{-4x}}{2} \Rightarrow 4 - \dfrac{3x}{2} = 2^{-4x-1}$
M1	Award the third M mark for adding x to **both** sides
M1	For simplifying and obtaining $(y) = x + 2^{-4x-1}$ or $(y) = x + 2^{-(4x+1)}$ on one side and a linear equation of the form $y = K \pm \dfrac{1}{2}x$ (may be unsimplified) where K is a constant, on the other side.
A1	For obtaining the correct equation $y = 4 - \dfrac{1}{2}x$ (may be unsimplified)
M1	For drawing their line correctly • Their line is in the form $y = k - \dfrac{x}{2}$, k nonzero constant (may be unsimplified) • There must be at least one intersection point with the given curve.
A1	Correct x values (both rounded to 1dp), ignore if $y = \ldots$ is present, accept also given as coordinates, even if their y coordinates are incorrect.

ALT I for first 5 marks

M1: Correctly removes log, achieves $2^{-4x} = 8 - 3x$

M1: For dividing by 2 throughout $\dfrac{2^{-4x}}{2} = \dfrac{8-3x}{2} \Rightarrow 2^{-(4x+1)} = 4 - \dfrac{3}{2}x$

M1: For adding x to both sides

M1: Rearranges leading to $x + 2^{-4x-1} = 4 - \dfrac{x}{2}$

A1: For obtaining the correct equation $y = 4 - \dfrac{1}{2}x$ (may be unsimplified)

ALT II for first 5 marks

M1: Correctly removes log, achieves $2^{-4x} = 8 - 3x$

M1: Rearrange the given curve to $y - x = 2^{-4x} \times 2^{-1}$

M1: Attempts to make 2^{-4x} the subject from $y - x = 2^{-4x} \times 2^{-1} \Rightarrow 2^{-4x} = \dots$

M1: Equates the two resulting equations, then attempts to make y=…

A1: For obtaining the correct equation $y = 4 - \dfrac{1}{2}x$ (may be unsimplified)

For reference the two correct equations are: $\begin{cases} 2^{-4x} = 8 - 3x \\ 2^{-4x} = 2y - 2x \end{cases} \Rightarrow 8 - 3x = 2y - 2x \Rightarrow y = 4 - \dfrac{x}{2}$

ALT III for first 5 marks

M1: Correctly removes log, achieves $2^{-4x} = 8 - 3x$

M1: For $2^{-(4x+1)} = 2^{-4x} \times 2^{-1}$ (may be embedded)

M1: Replaces 2^{-4x} with their $8 - 3x$

M1: Rearranges leading to y=…

A1: For obtaining the correct equation $y = 4 - \dfrac{1}{2}x$ (may be unsimplified)

Question	Scheme	Marks
7(a)	$$x = -\frac{5}{4}$$	B1 [1]
(b)	$$\left\{\frac{dy}{dx}\right\} = \frac{2x(4x+5) - 4(x^2-1)}{(4x+5)^2}$$ $$y = 0$$ $$\left\{\frac{dy}{dx}\right\} = \frac{2(-1)(4(-1)+5) - 4((-1)^2-1)}{(4(-1)+5)^2} = -2$$ $$\rightarrow \{gradient\ of\ normal =\} \frac{1}{2}$$ $$y - 0 = \frac{1}{2}(x - (-1)) oe$$	M1A1A1 B1 M1 M1A1cso [7]
(c)	$$\frac{x+1}{2} = \frac{x^2-1}{4x+5}$$ $$2x^2 + 9x + 7 = 0 \ \ OR \ \ (x+1)(4x+5) = 2(x+1)(x-1)$$ $$(2x+7)(x+1) = 0 \Rightarrow x = \cdots \ OR \ (4x+5) = 2(x+1) \Rightarrow x = \cdots$$ $$x = -\frac{7}{2}, \{x = -1\}$$ $$y = \frac{"-\frac{7}{2}" + 1}{2} = \cdots$$ $$\left(-\frac{7}{2}, -\frac{5}{4}\right)$$	M1 dM1 ddM1 A1 M1 A1 [6]
		Total 14 marks

Part	Mark	Notes
(a)	B1	$x = -\frac{5}{4}$ or $x = -1.25$, do not accept 4x+5=0
(b)	M1	Attempts to differentiate *y*. **Quotient Rule:** Look for $$\frac{Px(4x+5) - Q(x^2-1)}{(4x+5)^2} , where\ P > 0, Q > 0$$ If the quotient rule is quoted, it must be correct. Condone invisible bracket for this mark. **Product Rule:** Look for $Px(4x+5)^{-1} \pm Q(x^2-1)(4x+5)^{-2}$, $where\ P > 0, Q > 0$
	A1	Quotient rule: Correct denominator $(4x+5)^2$ and one correct term in the numerator, $2x(4x+5)$ or $-4(x^2-1)$. Product rule: One term correct
	A1	Fully correct. $\left\{\frac{dy}{dx} =\right\} \frac{2x(4x+5)-4(x^2-1)}{(4x+5)^2}$ or $\left\{\frac{dy}{dx} =\right\} 2x(4x+5)^{-1} - 4(x^2-1)(4x+5)^{-2}$ oe
	B1	*y* = 0 seen or implied
	M1	Correct method finding gradient of normal to the curve where *x*= -1
	M1	Correct method of forming the equation of a straight line using the point **(-1,their y value)** with a "**changed**" gradient.("changed" means a different gradient from their tangent gradient, their normal could be found by an incorrect method) If using *y=mx+c,* a value for *c* must be found and the equation formed.
	A1cso	For the correct line in any form, simplified or unsimplified. (isw) Once correct equation is seen, award the mark, ignore later incorrect simplification. e.g.2*y* = *x*+1 or 2y-x-1=0 or y=0.5x+0.5 oe or even $y - 0 = \frac{1}{2}(x - (-1))$
(c)	M1	For equating the equation of their line with the equation of the curve.
	dM1	For forming an equation with all like terms collected. **OR** cross multiplies then factorises e.g. 2($x^2 - 1$) = 2(x + 1)(x - 1)
	ddM1	Solves their equation with a correct method and finds at least one real value of *x* which is not -1, method of solving could be implied by correct answers if calculator used. **OR** cancels (*x*+1) and solves their linear equation.
	A1	$$x = -\frac{7}{2}, \qquad \{x = -1\}$$
	M1	For substituting their *x* value (not -1) to either the equation of their normal or the equation of the given curve to find a value for *y*
	A1	For the correct exact coordinates of $D\left(-\frac{7}{2}, -\frac{5}{4}\right)$ oe Allow x = -3.5, y = -1.25 instead of given as coordinates.

Question	Scheme	Marks
8(a)	$a + ar = 360$ oe e.g. $\dfrac{a(1-r^2)}{1-r} = 360$ $ar + ar^2 = 288$ oe $r = \left\{\dfrac{a(1+r)}{ar(1+r)} = \right\}\dfrac{288}{360} = \dfrac{4}{5}$ $a = \dfrac{360}{1 + "\frac{4}{5}"} = 200$ $U_n = 200\left(\dfrac{4}{5}\right)^{n-1}$ $*$	M1M1 (B1B1on ePen dM1A1 M1A1 A1cso(B1 on ePen) [7]
8(a) ALT	$a + ar = 360$ oe e.g. $\dfrac{a(1-r^2)}{1-r} = 360$ $ar + ar^2 = 288$ oe $a\left(\dfrac{360}{a} - 1\right) + a\left(\dfrac{360}{a} - 1\right)^2 = 288$ $\Rightarrow a = 200$ $r = \dfrac{360}{"200"} - 1 = \dfrac{4}{5}$ $U_n = 200\left(\dfrac{4}{5}\right)^{n-1}$ $*$	M1M1 (B1B1 on ePen) dM1A1 M1A1 A1cso(B1 on ePen) [7]
(b)	{S is convergent because} $\left\|\dfrac{4}{5}\right\| < 1$	B1 [1]
(c)	$S_\infty = \dfrac{'200'}{1 - '\left(\frac{4}{5}\right)'} = 1000$	M1A1cao [2]
(d)	$\dfrac{200\left(1 - \left(\frac{4}{5}\right)^n\right)}{1 - \left(\frac{4}{5}\right)} > 978$ $\left(\dfrac{4}{5}\right)^n < \dfrac{11}{500}$ $n \log\left(\dfrac{4}{5}\right) < \log\left(\dfrac{11}{500}\right)$ $\Rightarrow n > 17.104 \ldots$ $n = 18$	M1 dM1 ddM1 A1 [4]
		Total 14 marks

Part	Mark	Notes
(a)	M1	For either correct equation. Allow use of U_1 as the first term instead of a
	M1	For both correct equations. Allow use of U_1 as the first term instead of a
	dM1	For eliminating a from their equations, reaches $r =...$ (Depends on previous mark)
	A1	For the correct value of r
	M1	Uses their value of r to find a value for a
	A1	For $a = 200$
	A1 cso	For the correct required expression of $U_n = 200 \left(\frac{4}{5}\right)^{n-1}$ (must have U_n)
(a)ALT	M1	For either correct equation.
	M1	For both correct equations
	dM1	For eliminating r from their equations, reaches $a =...$ (Depends on previous mark)
	A1	For the correct value of a
	M1	Uses their value of a to find a value for r
	A1	For the correct value of r
	A1 cso	For the correct required expression of $U_n = 200 \left(\frac{4}{5}\right)^{n-1}$ (must have U_n)
(b)	B1	For stating the correct reason, $|r| < 1$ or $-1 < r < 1$ or "0.8" < 1 or $\left| "\frac{4}{5}" \right| < 1$ **Do not accept** just $r < 1$ without a correct reason
(c)	M1	For using their value of A /a /first term of their geometric sequence and their r (provided their $|r| < 1$)in a correct formula for the sum to infinity. $S_\infty = \frac{\text{'their A'}}{1 - \text{'(their r)'}}$
	A1cao	For the correct value of 1000
(d)	M1	Uses their a and r (their r can be $|r| < 1$ or $|r| > 1$) to set the correct formula for the sum of a geometric series > 978 or $=978$ or ≥ 978 (>978 or $=978$ or ≥ 978 may be implied by later work.)
	dM1	Attempts to rearrange and achieves $(\text{their } r)^n " < "k$ (k is non zero positive constant) **Allow strict or non-strict inequalities or equation**: $<, >, \geq, \leq$ or $=$
	ddM1	For correctly takes logs (any base)/ln, and achieves a positive value for n, it maybe implied by 17.1…but not 18 **ALT**: trial and error to find a value of n (at least tries $n=17$ and $n=18$)
	A1	For $n = 18$ Must come from **correct** working, achieves $n=18$ with no incorrect inequalities in their working.

If a candidate does not score the dM1 mark but arrives at an answer of 17.1...
this scores a maximum of M1 as clear use of calculator is not condoned.

Question	Scheme	Marks
9	**Area under the curve**	
	$A_c = \int_{-1}^{0} \left(-2e^{3x} + 4 \right) dx, = \left[\dfrac{-2e^{3x}}{3} + 4x \right]_{-1}^{0}$	M1M1
	$\Rightarrow \left(\dfrac{-2e^{3\times 0}}{3} + 4 \times 0 \right) - \left(\dfrac{-2e^{3\times -1}}{3} + 4 \times -1 \right), = \dfrac{10}{3} + \dfrac{2e^{-3}}{3}$	M1A1
	Area under trapezium	
	Method 1 [uses formula for the area of a trapezium]	
	$y = 2 \quad y = -2e^{-3} + 4$	
	$A_T = \dfrac{1}{2} \times 1 \times \left({}'2' + {}'-2e^{-3} + 4' \right) = \left[3 - e^{-3} \right]$	B1
	Method 2 [finds equation of the line and integrates]	M1
	Equation of the line: $y = \left(2e^{-3} - 2 \right)x + 2$ $\left(\text{accept } y = -1.9x + 2 \right)$	
	$A_T = \int_{-1}^{0} {}' \left(\left(2e^{-3} - 2 \right)x + 2 \right)' \, dx = \left[\dfrac{'\left(2e^{-3} - 2 \right)' x^2}{2} + \dfrac{'2'x}{2} \right]_{-1}^{0} = \left[3 - e^{-3} \right]$	
	Shaded Area	
	$A_{shaded} = \left(\dfrac{10}{3} + \dfrac{2e^{-3}}{3} \right) - \left(3 - e^{-3} \right) = \dfrac{1 + 5e^{-3}}{3}$	M1A1
		[8]
		Total 8 marks

Mark	Notes
M1	For a correct statement for the area under the curve with **correct limits**. This mark can be implied/embedded by later correct work. **Condone missing dx**
M1	Attempts to integrate the given curve to a form of $Ae^{3x} + Bx$ **Ignore limits for this mark**.
M1	For substituting in both of their limits into their integrated expression in the form $Ae^{3x} + Bx$ and subtracts. (A, B are nonzero constants) Correct exact answer $\dfrac{10}{3} + \dfrac{2e^{-3}}{3}$ implies the correct substitution. If their answer is not correct and exact, explicit substitution needs to be seen.
A1	For the correct **exact** area under the curve. If not seen explicitly, this can be implied by correct exact final answer.
Method 1 - trapezium	
B1	For both correct **exact** values of y
M1	For the correct method of finding area of the trapezium using their values for y Look for $\frac{1}{2} \times 1 \times (sum\ of\ their\ y\ values)$
Method 2 – equation of line	
B1	For the correct equation of the line L, must reach $y = \ldots$ $y = \left(2e^{-3} - 2\right)x + 2$ $\left(\text{accept } y = -1.9x + 2\right)$
M1	For the correct area under the line L using their line. Check for correct integration of their line and full substitution of -1 if their area is incorrect. (If the terms are equal to zero, substitution of 0 does not need to be seen.) Allow also $A_T = \int_{-1}^{0} {}'(-1.9x + 2)'\, dx = \left[\dfrac{'-1.9'x^2}{2} + \dfrac{'2'x}{2} \right]_{-1}^{0}$
Combines the areas	
M1	Correct strategy to find the value of shaded area The integrated area of the curve - the area of trapezium
A1	For the correct **exact** area of the shaded region in the required form
Some candidates are combining the curve – line in one calculation – mark this as follows.	

- Wherever you see the **correct** equation of the line, score the 5th Mark [B mark]
- Mark the work for the curve (which may be embedded in a combined integral) using the first 4 marks.
- Mark the area under the line using the 6th mark [M mark – using their equation in the form $y = mx + c$]
- When you see them subtracting the areas – award the penultimate M mark
 If they add the areas, this is M0
- Final A mark is for the correct answer only

Question	Scheme	Marks
10(a)	$$AC = \sqrt{(2x)^2 + x^2} = \left(\sqrt{5}x\right)$$ $$VA = '\sqrt{5}x'\sin 45^o = \frac{\sqrt{10}}{2}x \ *$$	B1 M1A1 cso [3]
(b)	$$\text{Height} = \sqrt{\left(\frac{\sqrt{10}}{2}x\right)^2 - \left(\frac{\sqrt{5}}{2}x\right)^2} = \frac{\sqrt{5}}{2}x$$	M1A1 [2]
(c)	$$cos \angle VBA = \frac{1}{\frac{\sqrt{10}}{2}}$$ $$\Rightarrow \angle VBA = 50.768...^o \approx 50.8^o \text{ awrt}$$	M1 A1 [2]
(d)	Let M be the point at which the diagonals AC and BD meet. The required angle is either AMB or DMC $AM = MB = \frac{AC}{2} = \frac{\sqrt{5}x}{2}$ or DM=MC=$\frac{AC}{2} = \frac{\sqrt{5}x}{2}$ $$cos \angle AMB = \frac{\left(\frac{\sqrt{5}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2 - 2^2}{2 \times \frac{\sqrt{5}}{2} \times \frac{\sqrt{5}}{2}} \ oe$$ $$\left(cos \angle AMB = -\frac{3}{5} \Rightarrow \angle AMB = 126.869...^o \right) \approx 126.9^o \text{ awrt}$$	B1 B1 M1 A1 [4]
(e)	$$9\sqrt{5} = \frac{1}{3} \times 2x \times x \times "\frac{\sqrt{5}}{2}x" \Rightarrow x = \cdots$$ $$x = 3$$	M1 A1 [2]
		Total 13 marks

Part	Mark	Notes
(a)	B1	For using correct Pythagoras theorem to find length *AC or ½ AC*, explicit substitution must be seen.
	M1	For correct trigonometry using their AC in triangle VAC or half of their AC in triangle VAM or correct Pythagoras theorem (let M be the centre of the base at which the diagonals *AC* and *BD* meet) to find length *VA*. *e.g.* $$VA = \sqrt{5}x\cos(45°) = \frac{\sqrt{10}}{2}x \quad or$$ $$VA = \frac{\sqrt{5}}{2}x \div \cos(45°) = \frac{\sqrt{10}}{2}x \quad or \quad VA = \frac{\sqrt{5}}{2}x \div \sin(45°) = \frac{\sqrt{10}}{2}x$$
	A1 cso	Achieves printed answer with no errors. Condone AV instead of VA Note: $\frac{\sqrt{10}}{2}x$ *written as* $\frac{\sqrt{10x}}{2}$ *is A0 ,* $\quad$ $\frac{\sqrt{10}}{2}x$ *written as* $\frac{\sqrt{10}x}{2}$ *is A1*
(b)	M1	Uses the correct Pythagoras theorem or correct trigonometry with the given length VA$=\frac{\sqrt{10}}{2}x$ to find the height of the pyramid.
	A1	For $h = \frac{\sqrt{5}}{2}x$ Correct exact answer only scores M1A1. This can be written down as the triangle is isosceles.
(c)	M1	**Ignore missing *x* consistently throughout their solution.** For using any appropriate trigonometry to find the angle *VBA* If attempts cosine rule, look for correct application of the cosine rule for the required angle: $$\left(\frac{\sqrt{10}}{2}x\right)^2 = \left(\frac{\sqrt{10}}{2}x\right)^2 + (2x)^2 - 2 \times \frac{\sqrt{10}}{2}x \times 2x \cos(VBA) \text{ oe}$$
	A1	For the correct angle. awrt 50.8 ° (with or without degree sign)
(d)	B1	Identifies the angle required.(can be implied/embedded in their working)
	B1	For the correct lengths AM and MB **or** DM and MC **or** BE and ME Let E to be mid point of AB, correct length BE=*x,* ME=0.5*x* scores B1, this mark can be implied/embedded in their working)
	M1	**Ignore missing *x* consistently throughout their solution.** For using any appropriate trigonometry to find the required angle or half of the required angle.
	A1	For the correct angle. awrt 126.9º (with or without degree sign)
(e)	M1	For using the correct formula for the volume of a pyramid with their height of the pyramid and attempts to find a value of *x*.
	A1	For $x = 3$ with or without unit, ignore incorrect unit

Question	Scheme	Marks
11(a)	$cos(A + A) = cos A \, cos A - sin A \, sin A$ $\Rightarrow cos 2A = cos^2 A - (1 - cos^2 A)$	M1
	$cos 2A = 2 cos^2 A - 1 *$	A1 cso [2]
(b)	$(2cos^2 A - 1)^2 = (cos2A)^2$	M1
	$cos 4A = 2cos^2 2A - 1 \Rightarrow cos^2 2A = \dfrac{cos4A + 1}{2}$	M1
	$(2cos^2 2A - 1)^2 = \dfrac{cos4A + 1}{2} *$	A1 cso [3]
(b) **ALT**	$cos 4A = 2 cos^2 2A - 1$ $= 2(2 cos^2 A - 1)^2 - 1$ $(2 cos^2 A - 1)^2 = \dfrac{cos 4A + 1}{2} *$	M1 M1 A1 cso [3]
(c)	$y = \dfrac{sin 2x}{2} + \dfrac{(2 cos^2 x - 1)^2}{2} + \dfrac{1}{8} \Rightarrow y = \dfrac{sin 2x}{2} + \dfrac{cos 4x}{4} + \dfrac{3}{8}$	M1
	$\dfrac{dy}{dx} = cos 2x - sin 4x$	dM1
	$\dfrac{dy}{dx} = cos 2x - 2 sin 2x \, cos 2x \{= 0\}$	B1
	$\{cos 2x(1 - 2 sin 2x) = 0\} \Rightarrow sin 2x = \dfrac{1}{2}$	
	$\Rightarrow x = \dfrac{sin^{-1}\left(\dfrac{1}{2}\right)}{2} = \cdots$	ddM
	$x = \dfrac{\pi}{12}$	A1
	$y = \dfrac{sin2\left("\dfrac{\pi}{12}"\right)}{2} + \dfrac{cos4\left("\dfrac{\pi}{12}"\right)}{4} + \dfrac{3}{8} = \dfrac{3}{4}\{\Rightarrow \left(\dfrac{\pi}{12}, \dfrac{3}{4}\right)\}$	M1A1 [7]
		Total 12 marks

Part	Mark	Notes
(a)	**M1**	For correct use of $cos(A + B) = cos A \cos B - \sin A \sin B \Rightarrow$ $cos(A + A) = \cos A \cos A - \sin A \sin A$ **OR** $\cos 2A = \cos^2 A - \sin^2 A$ **with** the Pythagorean identity to eliminate sine from the identity.
	A1 cso	Achieves printed answer with no errors seen. Left hand side must be cos2A not cos(A+A)
(b)	**M1**	Uses the given result from (a), replaces $(2\cos^2 A - 1)^2$ with $(\cos 2A)^2$
	M1	Uses the given result from (a) or a correct identity to replace $\cos^2 2A$ with $\frac{\cos 4A+1}{2}$
	A1 cso	Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.)
(b) ALT	**M1**	Uses the given result from (a) on cos 4A
	M1	Uses the given result from (a) or a correct identity on cos 2A
	A1 cso	Fully correct proof showing all necessary steps. (If they meet in the middle, they must have a conclusion, accept e.g. shown, proved, L=R, etc.)
(c)	**M1**	Rearranges the given y into the form $\frac{\sin 2x}{2} + \frac{\cos 4x+1}{k} + l$, oe, where k and l are nonzero constants Or $\frac{\sin 2x}{2} + \frac{\cos 4x}{P} + Q$, oe, where P and Q are nonzero constants (Allow mixed variables x and A for this method mark only)
	dM1	For an attempt to differentiate their y into the form $P\cos(2x) \pm Q\sin(4x)$ $oe, where\ P, Q\ are\ nonzero\ constants$ **Depends on the previous method mark.**
	B1	Uses $\sin 4x = 2\sin 2x \cos 2x$ or $\sin 4x = \sqrt{1-\cos^2 4x}$ oe in their dy/dx (may be implied by correct working)
	ddM1	Sets their differentiated expression $= 0$ (can be implied) and attempts a correct method to solve for $x=...$ **Depends on both previous method marks.**
	A1	For $x = \frac{\pi}{12}$, ignore extra x values are outside of given range. Withhold A mark if extra solution seen in the given range.
	M1	For attempting to find the value of y using their x, found by solving their $\frac{dy}{dx} = 0$, provided x is in the range $0 \leqslant x \leqslant \frac{\pi}{6}$ Substitutes into the given y **or** their y, correct answer implies this mark, but for incorrect answer, must see explicit substitution
	A1	For the correct exact coordinates of P, $\left(\frac{\pi}{12}, \frac{3}{4}\right)$ or $\left(\frac{\pi}{12}, 0.75\right)$, accept $x = \frac{\pi}{12}$, $y = \frac{3}{4}$ or $x = \frac{\pi}{12}$, $y = 0.75$