Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International GCSE

## Friday 7 June 2024

Morning (Time: 2 hours)

**Paper reference** **4PM1/02**

# Further Pure Mathematics
## PAPER 2

**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

P76508A


F:1/1/1/1/1/

P 7 6 5 0 8 A 0 1 3 2

Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

### Series

**Arithmetic series**

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

**Geometric series**

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

### Calculus

**Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

### Trigonometry

**Cosine rule**
In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

### Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** $f(x) = 6x^3 - 13x^2 + ax - 10$ where $a$ is a constant

Given that $(3x - 2)$ is a factor of $f(x)$

(a) show that $a = 21$

**(2)**

(b) Hence show algebraically that the curve $y = f(x)$ has only one intersection with the $x$-axis.

**(4)**

**(Total for Question 1 is 6 marks)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**2** The quadratic equation $3x^2 - 5x + 1 = 0$ has roots $\alpha$ and $\beta$

Without solving the equation,

form a quadratic equation with integer coefficients, that has roots $\frac{\alpha}{2\beta}$ and $\frac{\beta}{2\alpha}$

**(8)**

**Question 2 continued**

**(Total for Question 2 is 8 marks)**

**3**

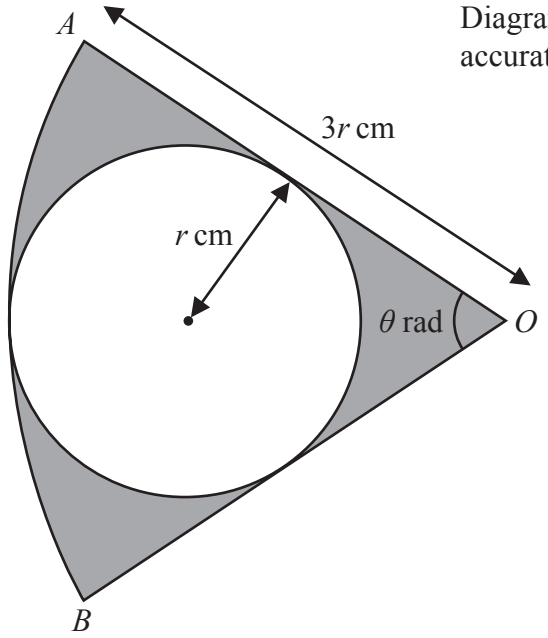


Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows the sector $AOB$ of a circle with centre $O$ and radius $3r$ cm

A circle with radius $r$ cm touches $OA$ and $OB$ and the arc $AB$

Angle $AOB$ is $\theta$ radians, where $0 < \theta < \frac{\pi}{2}$

(a) Find the exact value of $\theta$

**(2)**

The area of the region shown shaded in Figure 1 is $8\pi$ cm$^2$

(b) Find the value of $r$

**(4)**

**Question 3 continued**

**(Total for Question 3 is 6 marks)**

**4**

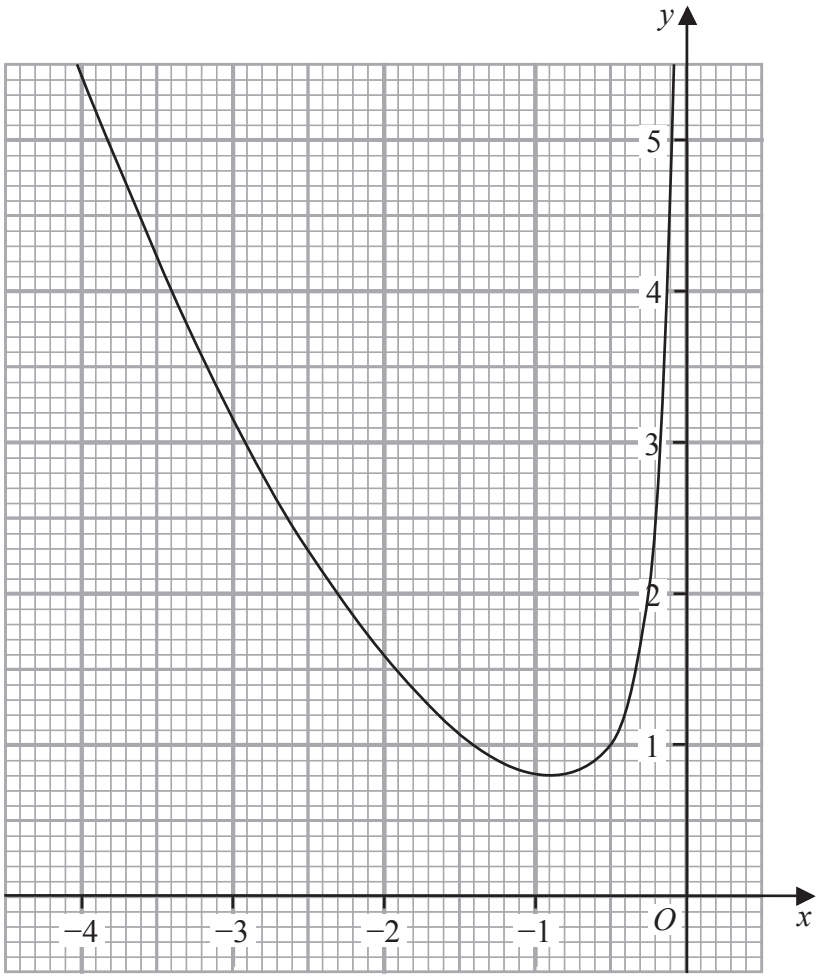


**Figure 2**

Figure 2 shows part of the curve with equation $y = \frac{x^2}{3} - \frac{1}{2x}$ for $-4 < x < 0$

By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 + 3x^2 - 36x - 6 = 0$ in the interval $-4 < x < 0$

**(4)**

**Question 4 continued**

**(Total for Question 4 is 4 marks)**

**5**

$$y = e^{2x}\left(x^2 - 5x\right)$$

Show that $2e^{2x} = \dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y$

**(7)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 5 continued**

**(Total for Question 5 is 7 marks)**

**6**

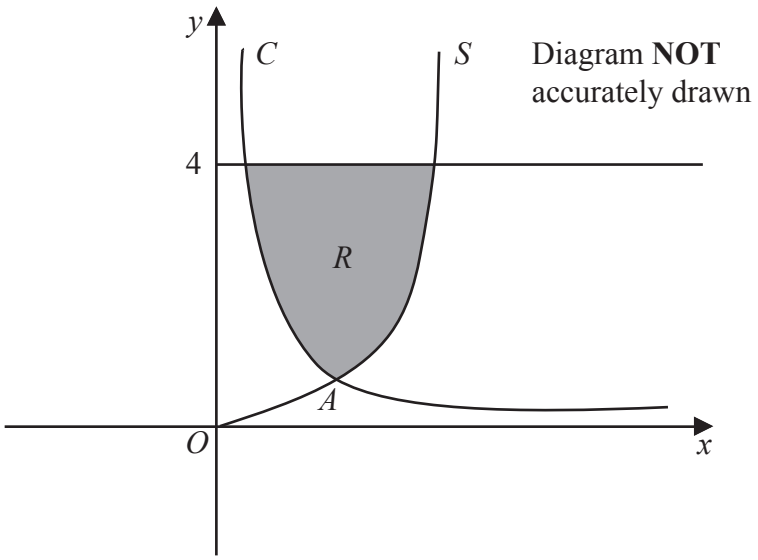


**Figure 3**

Figure 3 shows part of the curve $C$ with equation $y = \frac{1}{4x}$, $x > 0$ and part of the curve $S$ with equation $y = 2x^2, x \geqslant 0$

The curve $C$ and the curve $S$ intersect at the point $A$

(a) Find the coordinates of point $A$

**(3)**

The finite region $R$, shown shaded in Figure 3, bounded by the curve $C$, the curve $S$ and the straight line $y = 4$ is rotated through 360° about the $y$-axis.

(b) Find, using algebraic integration, the exact volume of the solid formed.

**(7)**

**Question 6 continued**

**(Total for Question 6 is 10 marks)**

**7** (a) Expand $\left(1 + 2x^2\right)^{-\frac{3}{4}}$ in ascending powers of $x$ up to and including the term in $x^6$

Express each coefficient as an exact fraction in its lowest terms.

**(3)**

$$f(x) = \frac{(2 + kx)}{\left(1 + 2x^2\right)^{\frac{3}{4}}} \quad \text{where } k \neq 0$$

(b) Obtain a series expansion for $f(x)$ in ascending powers of $x$ up to and including the term in $x^5$

Give each coefficient in terms of $k$ where appropriate.

**(2)**

The coefficient of the term in $x^5$ is fourteen times the coefficient of the term in $x^2$

(c) Find the value of $k$

**(2)**

**Question 7 continued**

**Question 7 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 7 continued**

**(Total for Question 7 is 7 marks)**

**8**

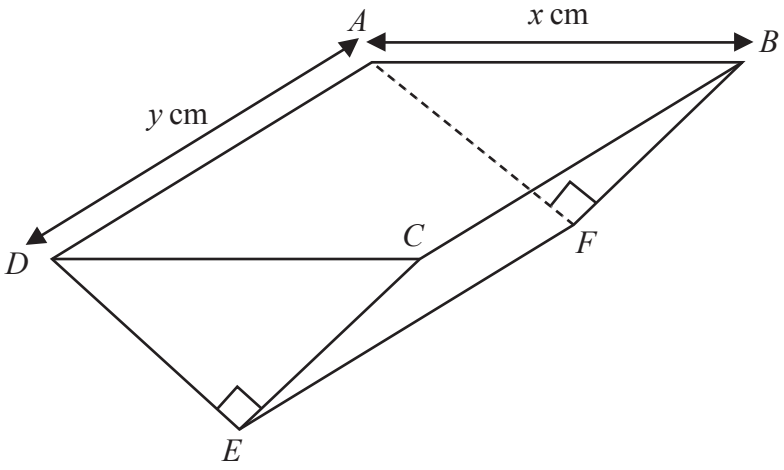


Diagram **NOT** accurately drawn

**Figure 4**

Figure 4 shows a solid right triangular prism $ABCDEF$

The cross section of the prism is an isosceles triangle.

- $\angle DEC = \angle AFB = 90^{\circ}$
- $AB = DC = x$ cm
- $AD = BC = FE = y$ cm
- $AF = BF = DE = CE$

The triangular faces of the prism are vertical and the edges $AD$, $BC$ and $FE$ are horizontal.
The volume of the prism is 3.6 $cm^3$
The total external surface area of the prism is $S$ $cm^2$

(a) Show that $S$ satisfies the equation

$$S = \frac{x^2}{2} + \frac{72\left(\sqrt{2}+1\right)}{5x}$$

**(4)**

Given that $x$ can vary,

(b) use calculus, to find to 3 significant figures, the value of $x$ for which $S$ is a minimum.
Justify that this value of $x$ gives a minimum value of $S$

**(4)**

(c) Hence find, to 2 significant figures, the minimum value of $S$

**(2)**

.....

.....

.....

**Question 8 continued**

**Question 8 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 8 continued**

**(Total for Question 8 is 10 marks)**

**9**

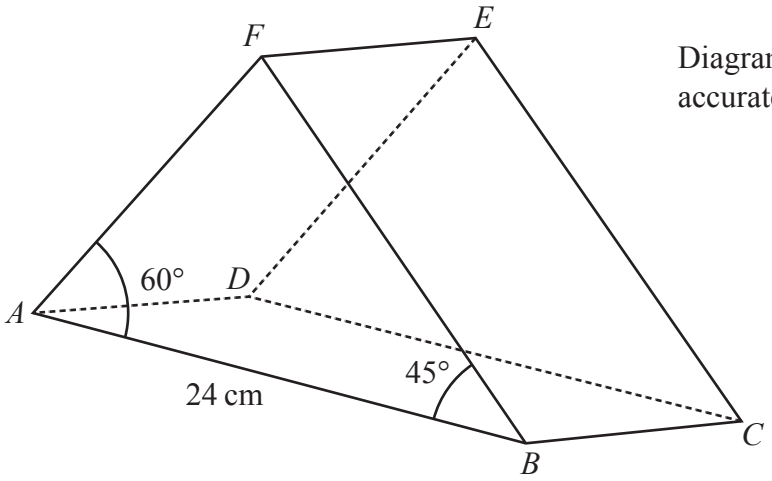


Diagram **NOT** accurately drawn

**Figure 5**

Figure 5 shows a right triangular prism $ABCDEF$ where $ABCD$ is a rectangle.

$AF = DE$ $BF = CE$ $AD = FE = BC$ $AB = DC = 24$ cm

$\angle ABF = \angle DCE = 45°$ $\angle BAF = \angle CDE = 60°$

Using a formula from page 2,

(a) show that $\sin AFB = \dfrac{\sqrt{2} + \sqrt{6}}{4}$

**(3)**

Without using a calculator,

(b) show that $BF = 12\left(3\sqrt{2} - \sqrt{6}\right)$ cm

**(5)**

The angle between the plane $AEB$ and the plane $ABCD$ is $65°$

(c) Find, in cm to 2 significant figures, the length of $EF$

**(3)**

(d) Find, in degrees to one decimal place, the size of the angle between the line $CF$ and the plane $ABCD$

**(4)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 9 continued**

**Question 9 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 9 continued**

**(Total for Question 9 is 15 marks)**

**10** The points $A$, $B$, $C$ and $D$ are the vertices of a quadrilateral such that

$$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b} \quad \overrightarrow{AC} = 7\mathbf{a} + 9\mathbf{b} \quad \overrightarrow{AD} = 4\mathbf{a} + 5\mathbf{b}$$

(a) Show that $ABCD$ is a parallelogram.

**(3)**

$BC$ is extended to the point $E$ such that $BCE$ is a straight line.

Point $F$ lies on $CD$ such that $CF : FD = 1 : 2$

Given that $A$, $F$ and $E$ are collinear,

(b) find the vector $\overrightarrow{AE}$ in the form $X\mathbf{a} + Y\mathbf{b}$ where $X$ and $Y$ are rational numbers to be found.

**(8)**

**Question 10 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 10 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Question 10 continued**

**(Total for Question 10 is 11 marks)**

**11** Using formulae from page 2, show that

(a) (i) $\cos 2A = 2\cos^2 A - 1$

**(3)**

(ii) $\sin 2A = 2\sin A \cos A$

**(1)**

(b) Show that $\cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$

**(4)**

Hence, or otherwise,

(c) solve, giving exact values in terms of $\pi$

$$8\cos^3\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \qquad \text{for } 0 \leqslant \theta \leqslant 2\pi$$

**(4)**

(d) use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(4\cos^3\theta - \sin 2\theta\right) \mathrm{d}\theta$$

**(4)**

**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 16 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA