Mark Scheme (Results)

Summer 2024

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - cso – correct solution only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$,where $|pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where$|pq| = |c|$ and $|mn| = |a|$leading to $x = \ldots$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading ccto $x = \ldots$

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values **or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Marks
1	**Angle** $(4x)^2 = (2x)^2 + (3x)^2 - 2\times 2x\times 3x\times \cos ABC$ **or** $\left(\cos ABC =\right)\dfrac{(2x)^2+(3x)^2-(4x)^2}{2\times 2x\times 3x} \Rightarrow \angle ABC = 104.47751...^{\circ}(1.823476...^{c})$	M1A1
	OR $(3x)^2 = (2x)^2 + (4x)^2 - 2\times 2x\times 4x\times \cos BAC$ **or** $\left(\cos BAC =\right)\dfrac{(2x)^2+(4x)^2-(3x)^2}{2\times 2x\times 4x} \Rightarrow \angle BAC = 46.56746...^{\circ}(0.812755...^{c})$	[M1A1]
	OR $(2x)^2 = (3x)^2 + (4x)^2 - 2\times 3x\times 4x\times \cos ACB$ **or** $\left(\cos ACB =\right)\dfrac{(3x)^2+(4x)^2-(2x)^2}{2\times 3x\times 4x} \Rightarrow \angle ACB = 28.95502...^{\circ}(0.505360....^{c})$	[M1A1]
	Area $50 = \dfrac{1}{2}\times 2x\times 3x\times \sin"104.47751"^{\circ} \Rightarrow x^2\left(=17.213259...\right)$	dM1
	$\Rightarrow x = 4.15$	A1
	OR $50 = \dfrac{1}{2}\times 2x\times 4x\times \sin"46.56746"^{\circ} \Rightarrow x^2\left(=17.213259...\right)$	[dM1
	$\Rightarrow x = 4.15$	A1]
	OR $50 = \dfrac{1}{2}\times 3x\times 4x\times \sin"28.95502"^{\circ} \Rightarrow x^2\left(=17.213259...\right)$ $\Rightarrow x = 4.15$	[dM1 A1] [4]
ALT1	Uses Heron's formula $s = \dfrac{2x+3x+4x}{2} = \dfrac{9x}{2}$ oe	M1A1
	$50 = \sqrt{"\dfrac{9x}{2}"\left("\dfrac{9x}{2}"-2x\right)\left("\dfrac{9x}{2}"-3x\right)\left("\dfrac{9x}{2}-4x"\right)} = \sqrt{\dfrac{135x^4}{16}}$	dM1
	$\Rightarrow x^2 = 17.213259... \Rightarrow x = 4.15$	A1 [4]

ALT 2	$(4x)^2 = (2x)^2 + (3x)^2 - 2\times 2x\times 3x\times \cos ABC$ **or** $\left(\cos ABC =\right)\dfrac{(2x)^2+(3x)^2-(4x)^2}{2\times 2x\times 3x}\left(=-\dfrac{1}{4}\right)$ $\Rightarrow \left(\sin ABC =\right)\sqrt{\dfrac{15}{16}}\left(=\dfrac{\sqrt{15}}{4}=0.9682458...\right)$ oe OR $(3x)^2 = (2x)^2 + (4x)^2 - 2\times 2x\times 4x\times \cos BAC$ **or** $\left(\cos BAC =\right)\dfrac{(2x)^2+(4x)^2-(3x)^2}{2\times 2x\times 4x}\left(=\dfrac{11}{16}\right)$ $\Rightarrow \left(\sin ABC =\right)\sqrt{\dfrac{135}{256}}\left(=3\dfrac{\sqrt{15}}{16}=0.7261843...\right)$ oe OR $(2x)^2 = (3x)^2 + (4x)^2 - 2\times 3x\times 4x\times \cos ACB$ **or** $\left(\cos ACB =\right)\dfrac{(3x)^2+(4x)^2-(2x)^2}{2\times 3x\times 4x}\left(=\dfrac{7}{8}\right)$ $\Rightarrow \left(\sin ABC =\right)\sqrt{\dfrac{15}{64}}\left(=\dfrac{\sqrt{15}}{8}=0.4841229...\right)$ oe $50=\dfrac{1}{2}\times 2x\times 3x\times "\sqrt{\dfrac{15}{16}}" \Rightarrow x^2\left(=17.213259...\right)$ or $x=$ $\Rightarrow x = 4.15$ OR $50=\dfrac{1}{2}\times 2x\times 4x\times "\sqrt{\dfrac{135}{256}}" \Rightarrow x^2\left(=17.213259...\right)$ or $x=$ $\Rightarrow x = 4.15$ OR $50=\dfrac{1}{2}\times 3x\times 4x\times "\sqrt{\dfrac{15}{64}}" \Rightarrow x^2\left(=17.213259...\right)$ or $x=$ $\Rightarrow x = 4.15$	M1 A1 {M1 A1} {M1 A1} dM1 A1 {dM1 A1} {dM1 A1} [4]
		Total 4 marks

<table>
<tr><th>Mark</th><th>Notes</th></tr>
<tr><td>M1</td><td>For a fully correct substitution into the cosine formula as shown. ie including an equals sign.
OR for a fully correct expression for cosABC or cosBAC or cosACB
Allow students to use just B, A and C for angles or any labelling.</td></tr>
<tr><td>A1</td><td>For one of the correct angles in the triangle.</td></tr>
<tr><td colspan="2">Note for M1 A1
In this question, we will override the general principle of marking for multiple attempts, if the attempts are finding other angles. Mark the attempt that is correct.
The x can be consistently omitted or
the x can be recovered (also indicated by a correct angle) or
values in the correct proportions can be used as an alternative (also indicated by a correct angle).
M1 may also be awarded if the values in the expression or equation lead to the correct angle for those values, regardless of the labelling of the angle. This may also lead to A1
$(2x)^2 = (3x)^2 + (4x)^2 - 2\times 3x\times 4x\times \cos A$ or
e.g.
$\left(\cos A =\right)\dfrac{(3x)^2+(4x)^2-(2x)^2}{2\times 3x\times 4x} \Rightarrow \angle A = 28.95502...^{\circ}$
All the values here are correct to give 28.95502, but the labelling of angle A is incorrect.
Allow angles in degrees to be rounded to the nearest whole number here and angles in radians to be rounded to 1 dp.
Where candidates do not actually work the angle out, if and only if the fully correct expression for cos(angle) or angle = $\cos^{-1}$ is seen in the 1st M1, look for any of the following used in the 2nd M1, then this A mark can be awarded.
.
$\sin\left(\cos^{-1}(\text{ANS})\right)$ or $\sin\left(\cos^{-1}\left(\left((\mathit{angle})\ A\right)\right)\right)$ or $\sin\left(\cos^{-1}\left(\left((\mathit{angle})\ B\right)\right)\right)$
or $\sin\left(\cos^{-1}\left(\left((\mathit{angle})\ C\right)\right)\right)$
or for $\sin\left(\cos^{-1}\left(-\dfrac{1}{4}\right)\right)$ or $\sin\left(\cos^{-1}\left(\dfrac{11}{16}\right)\right)$ or $\sin\left(\cos^{-1}\left(\dfrac{7}{8}\right)\right)$ seen</td></tr>
<tr><td>dM1</td><td>For using the correct formula for the area of a triangle using 50 and their angle (or use of sin$\cos^{-1}$ correctly and the correct 2 sides for their angle and a fully correct rearrangement to find x^2 or x
If 1st M1 has been gained and a poor rearrangement occurs to give an incorrect angle ft their angle. This is a general principle of marking.
Sight of 17.2(1……….) following M1 A1 and the correct formula written will imply this mark.
The same condition applies, if the angle is not actually worked out as for the A mark.
Dependent on previous method mark.</td></tr>
<tr><td>A1</td><td>For the correct value of x. Accept awrt 4.15 [cm]</td></tr>
<tr><td>ALT1</td><td></td></tr>
<tr><td>M1</td><td>For fully correct substitution, as shown into Heron's formula.</td></tr>
<tr><td>A1</td><td>For the correct expression for s</td></tr>
<tr><td colspan="2">x must be present, but may be recovered (indicated by A1)</td></tr>
<tr><td>dM1</td><td>Applies Heron's formula correctly with their value of s and 50 and a fully correct rearrangement to find x^2 or x</td></tr>
<tr><td>A1</td><td>For the correct value of x. Allow all awrt 4.15.</td></tr>
</table>

ALT2	
M1	For a fully correct substitution into the cosine **formula** as shown. ie including an equals sign. OR for a fully correct **expression** for cos*ABC* or cos*BAC* or cos*ACB* Allow students to use just *B*, *A* and *C* for angles or any labelling.
A1	For one of the correct values for sin(relevant angle)
Note for M1 A1 The x can be consistently omitted **or** the x can be recovered (also indicated by a correct value for sin) **or** values in the correct proportions can be used as an alternative (also indicated by a correct value for sin). M1 may also be awarded if the values in the expression or equation lead to the correct sin value for those values, regardless of the labelling of the angle. This may also lead to A1 e.g. $(2x)^2 = (3x)^2 + (4x)^2 - 2\times 3x\times 4x\times \cos A$ **or** $\left(\cos A =\right)\dfrac{(3x)^2+(4x)^2-(2x)^2}{2\times 3x\times 4x} \Rightarrow \sin A = \sqrt{\dfrac{15}{64}}$ All the values here are correct to give $\Rightarrow \left(\sin ABC =\right)\sqrt{\dfrac{15}{64}}$, but the labelling of angle *A* is incorrect. Allow values to be rounded to 1 dp.	
dM1	For using the correct formula for the area of a triangle using 50 and their sin value correctly and the correct 2 sides for their angle and a fully correct rearrangement to find x^2 or x If M1 has been gained but their sin value is incorrect, students must be seen to have substituted into $\cos^2(\text{angle})+\sin^2(\text{angle})=1$. Though a poor rearrangement to give an incorrect sin value, will potentially allow this mark to be awarded. Sight of 17.2(1……….) following M1 A1 will imply this mark. Dependent on previous method mark.
A1	For the correct value of *x*. Accept awrt 4.15 [cm]

Question	Scheme	Marks
For part (a) of this question, mark using the scheme which gives the most marks.		
2 (a)	$A=2 \quad B=1 \quad C=7$	B1B1B1
ALT	$2x^2+4x+9=2\left(x^2+2x\right)+9=2\left[\left(x+1\right)^2-1\right]+9$ $\Rightarrow \mathrm{f}\left(x\right)=2\left(x+1\right)^2+7 \qquad A=2 \quad B=1 \quad C=7$	M1M1 A1 [3]
(b)(i)	-1	B1ft
(ii)	$\frac{1}{7}$	B1ft [2]
		Total 5 marks

Part	Mark	Notes
• **Mark using the B scheme first.** • If not full marks – use the MMA scheme also, if appropriate. • Same score – apply the B marks. • Higher score – apply the marks from the MMA scheme. Allow the candidate to state *A*, *B* and *C* or the values to be embedded within an expression.		
(a) Different marks on ePen	B1	For one of *A*, *B*, or *C* correct.
	B1	For two of *A*, *B*, or *C* correct.
	B1	For all of *A*, *B*, and *C* correct.
ALT	M1	For correctly factorising the given expression to achieve either: $2\left(x^2+2x\right)+9 \quad \text{or} \quad 2\left(x^2+2x+\frac{9}{2}\right)$
	M1	Completes the square correctly, **regardless of any factor on the outside – follow through their factorisation.** $\left(x^2+ax+b\right) \ \text{or} \ \left(x^2+ax\right)+c \Rightarrow$ ie $\left[\left(x+\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2+b\right]$ or $\left[\left(x+\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2\right]+c \qquad a,b,c\neq 0$
	A1	For all of *A*, *B* and *C* correct.
Note, the question does not ask students to show working nor preclude the use of a calculator, so if values are simply listed, these can be given marks from the B scheme throughout.		
(b)(i)	B1ft	For the correct value or follow through their –*B*
(ii)	B1ft	For the correct value or follow through their $\frac{1}{C}$
Only if no labelling of (i) and (ii)	SC2	If candidate clearly writes max value of $\frac{1}{7}$ when $x = -1$. Allow ft
	SC1	If candidate clearly writes $\left(-1,\frac{1}{7}\right)$ Allow ft Marked as 1st B1
If no labelling of (i) and (ii) is present for parts (b) – marks may be awarded for the values presented in the correct order. **B0 B0 if not labelled and work doesn't meet this condition.**		

Question	Scheme	Marks
3(a)	$\left(\sum_{r=1}^{n}(5r-3)\Rightarrow\right) a=2,\quad d=5$	B1
	$\left(\sum_{r=1}^{n}(5r-3)=\right)\frac{n}{2}(2\times 2+(n-1)5)$ or $\frac{n}{2}(4+5n-5)$ or $\frac{n}{2}(2+2+(n-1)5)$ or $\frac{n}{2}(2+5n-3)$	M1
	$=\frac{n}{2}(5n-1)*$	A1 cso [3]
	ALT (Using standard results) $\left(\sum_{r=1}^{n}(5r-3)=\right)5\sum_{1}^{n}r-3\sum_{1}^{n}1$	B1
	$\left(\sum_{r=1}^{n}(5r-3)=\right)5\left[\frac{n}{2}(n+1)\right]-3n=\frac{5n^2+5n-6n}{2}=\frac{n}{2}(5n-1)*$	M1A1 cso [3]
(b)	$\left(\sum_{31}^{60}(5r-3)=\right)\frac{60}{2}(5\times 60-1)-\frac{30}{2}(5\times 30-1)=6735$	M1A1 [2]
	ALT $\left(\sum_{31}^{60}(5r-3)=\right)\frac{30}{2}(152+297)=6735$ or $\left(\sum_{1}^{60}(5r-3)-\sum_{1}^{30}(5r-3)=\right)\frac{60}{2}(2\times 2+(60-1)\times 5)-\frac{30}{2}(2\times 2+(30-1)\times 5)=6735$	M1A1 [2]
(c)	$\frac{n}{2}(5n-1)=3783\Rightarrow 5n^2-n-7566=0$	M1
	$\Rightarrow(5n+194)(n-39)=0$	M1
	$\Rightarrow n=39\quad\left[-\frac{194}{5}\right]$	A1 [3]
		Total 8 marks

Part	Mark	Notes
(a)	B1	For the values of *a* and *d*, these may be explicitly stated or implicitly used in a formula.
	M1	Correctly substitutes their values of *a* and *d* into the correct summation formula
	A1 cso	For the correct expression, minimum steps as shown, no errors or omissions.
	ALT	
	B1	For writing the given expression as $5\sum_{1}^{n} r - 3\sum_{1}^{n} 1$
	M1	For the sum shown, using standard results for the series.
	A1 cso	For the correct expression, minimum steps as shown, no errors or omissions.
(b)	M1	Uses the given expression with both n = 60 and n = 30 and subtracts. Indicated by 8970 – 2235. As a concession, allow substitution of $n = 31$ and 60 for the mark. **ALT** Correct substitution of the correct ‘first’ and ‘last’ value into the correct summation formula ie $\frac{30}{2}(152+297)$. Allow as a concession $\frac{29}{2}(152+297)$ or correct substitution of both n = 30 and n = 60 into the correct sum to n terms formula and a subtraction. Indicated by 8970 – 2235. As a concession, allow substitution of $n = 31$ and 60 for the mark.
	A1	For 6735
(c)	M1	For correctly placing the given expression = 3783 and rearranging (allow one error) to get a 3TQ = 0.
	M1	For solving their 3TQ and achieving at least one value. Minimum attempt to solve required (see general guidance). A correct value of 39 can imply this mark
	A1	For 39 only. If $-\frac{194}{5}$ is also given as a solution, withhold this mark.

Question	Scheme	Marks
4	$\frac{dA}{dt} = 50\pi \qquad \frac{dr}{dt} = \frac{5}{12}$	B1,B1
	$\left(A = 4\pi r^2 \Rightarrow \frac{dA}{dr} = \right) 8\pi r$	B1
	$\frac{dr}{dt} = \frac{1}{\frac{dA}{dr}} \times \frac{dA}{dt}$ **oe** $\Rightarrow \frac{5}{12} = \frac{1}{8\pi r} \times 50\pi \Rightarrow r = 15$	M1dM1A1
	$V = \frac{4}{3} \times \pi \times 15^3 = 4500\pi \left[\text{cm}^3\right]$	M1A1
		[8]
		Total 8 marks

Mark	Notes
B1	For either the correct $\frac{dA}{dt} = 50\pi$ or $\frac{dr}{dt} = \frac{5}{12}$ May be seen explicitly or used implicitly in the candidate's work.
B1	For both correct $\frac{dA}{dt} = 50\pi$ and $\frac{dr}{dt} = \frac{5}{12}$ May be seen explicitly or implicitly in the candidate's working.
B1	For $8\pi r$ May be seen explicitly or implicitly in the candidate's working.
M1	For a correct chain rule, relevant to the question. Look for equivalences. This may be explicitly stated or can be awarded for use of the appropriate values or expressions implicitly.
dM1	For correctly substituting their values and rearranging their equation to find a value for r. Allow errors in rearrangement. Dependent on the previous method mark, though M1 dM1 can be awarded if the implicit use of their values in a correct chain rule is correct, without the chain rule being stated.
A1	For r = 15 (cm)
M1	For using the formula for the volume of a sphere with their r **Note – although this has not been made a dependent mark, this mark can only be awarded if 'their *r*' has come from some attempt at calculus.**
A1	For the correct volume of a sphere given exactly.

Question	Scheme	Marks
5(a)	$\left(v = \int (3t-4)\ \mathrm{d}t =\right) \frac{3t^2}{2} - 4t + c$	M1
	(When $t = 0$, $v = 0 \Rightarrow c = 0$)	
	$(v =) \frac{3\times 4^2}{2} - 4\times 4 = 8$ [m/s]	M1A1 [3]
(b)	$0 = \frac{3t^2}{2} - 4t \Rightarrow t\left(\frac{3t}{2} - 4\right) = 0 \Rightarrow \frac{3t}{2} = 4 \Rightarrow t = \frac{8}{3}$	M1A1 [2]
(c)	$\left(x = \int\left(\frac{3t^2}{2} - 4t\right)\mathrm{d}t =\right) \frac{3t^3}{2\times 3} - \frac{4t^2}{2}(+c) = \left[\frac{t^3}{2} - 2t^2 + c\right]$	M1
	When $t = 0$ P is at the point with coordinates (– 10, 0) $\Rightarrow c = -10$	M1
	$\Rightarrow (x =) \frac{3^3}{2} - 2\times 3^2 - 10 = -\frac{29}{2}$ (m)	dM1A1 [4]
		Total 9 marks

Part	Mark	Notes
(a)	M1	For a minimally acceptable attempt (see general guidance) to integrate the given expression. **At least one term correctly integrated.** Terms do not need to be simplified to attain this mark. No power of t to decrease.
	M1	For substituting the value of $t = 4$ into their changed expression (general rule of marking unless precluded, this method mark may be implied from a correct answer, unless from incorrect working).
	A1	For $(v =)8$
We do not need to see the calculation of $c = 0$ Note: substitution of $t = 4$ into the expression for a also gives an answer of 8 – please watch out for this and do not mark this correct.		
(b)	M1	For setting their changed expression = 0 and a **fully correct** attempt to solve, leading to a value for t. Note a correct vale of t can imply this mark. If the quadratic is not the correct quadratic, method must be shown.
	A1	For the value of $t = \frac{8}{3}$, ignore $t = 0$ is given. Accept answers which round to 2.7 or clear indication of 2.6 recurring.
(c)	M1	For a **minimally acceptable attempt** (see general guidance) to integrate their expression for v, which must be a **minimum 2 term** expression. Terms don't need to be simplified at this point. c does not need to be present. No power of t to decrease.
	M1	For correct substitution, into their changed expression, of $t = 0$ and $P = -10$ to find the value of c. – 10, if seen at any point will usually imply this mark. The mark is for substitution of the correct values, to find c, if they rearrange incorrectly, the mark can still be awarded.
	dM1	For substituting the value of $t = 3$ into their changed expression, with their c. Dependent on the previous method mark.
	A1	For the displacement of $-\frac{29}{2}$ If distance of $\frac{29}{2}$ is given A0, no isw here.

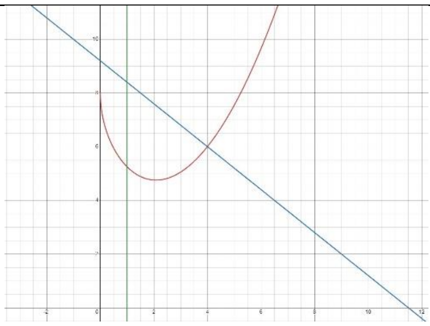
Question	Scheme	Marks
6 (a)	$p=1,\quad q=8$	B1B1 [2]
(b)	$\left(\text{Gradient of } l=\frac{12-2}{3--2}=\right)2$ (Gradient of the perpendicular =) $-\frac{1}{"2"}$ Equation of k $y-"8"="-\frac{1}{2}"\left(x-"1"\right)$ $\Rightarrow 2y+x-17=0\quad *$	B1 B1ft M1 A1 cso [4]
(c)	When y = 0, $\left(2\times 0+x-17=0\Rightarrow\right)x=17$ $\left(\lvert CD\rvert=\right)\sqrt{\left(0-"8"\right)^2+\left("17"-"1"\right)^2}=\sqrt{320}=\left[8\sqrt{5}\right]$	 B1 M1A1 [3]
(d)	(Length of CX =) $\frac{2\times 80}{8\sqrt{5}}\left(=4\sqrt{5}\right)$ $\left("4\sqrt{5}"\right)^2=\left(m-"1"\right)^2+\left(n-"8"\right)^2$ $"2"=\frac{n-"8"}{m-"1"}\Rightarrow\left[n=2m+6\right]$ oe **or** $y-2="2"(x+2)\Rightarrow\left[y=2x+6\right]$ **or** $y-12="2"\left(x-3\right)\Rightarrow\left[y=2x+6\right]$ oe $\Rightarrow "80"=\left(m-"1"\right)^2+\left(2m+6-"8"\right)^2\left(\Rightarrow 0=5m^2-10m-75 \text{ oe eg } 0=m^2-2m-15\right)$ $\Rightarrow\left(5m+15\right)\left(m-5\right)=0$ oe eg $\left(m+3\right)\left(m-5\right)=0$ $m=5,\left(-3\right)$ $n\left(=2m+6=2\times 5+6\right)=16$	B1 M1 M1 ddM1 M1 A1 A1 [7]
ALT1	(Length of CX =) $\frac{2\times 80}{8\sqrt{5}}\left(=4\sqrt{5}\right)$ $\left(m-"17"\right)^2+\left(n\right)^2=\left("4\sqrt{5}"\right)^2+\left("8\sqrt{5}"\right)^2$ $"2"=\frac{n-"8"}{m-"1"}\Rightarrow\left[n=2m+6\right]$ oe **or** $y-2="2"(x+2)\Rightarrow\left[y=2x+6\right]$ **or** $y-12="2"\left(x-3\right)\Rightarrow\left[y=2x+6\right]$ oe $\Rightarrow "400"=\left(m-"17"\right)^2+\left(2m+6\right)^2\left(\Rightarrow 0=5m^2-10m-75 \text{ oe eg } 0=m^2-2m-15\right)$ $\Rightarrow\left(5m+15\right)\left(m-5\right)=0$ oe eg $\left(m+3\right)\left(m-5\right)=0$ $m=5,\left(-3\right)$ $n\left(=2m+6=2\times 5+6\right)=16$	B1 M1 M1 ddM1 M1 A1 A1 [7]

ALT2	(Length of CX =) $\dfrac{2\times 80}{8\sqrt{5}}\left(=4\sqrt{5}\right)$	B1
	Length of AB = $\sqrt{(12-2)^2+(3--2)^2}\ \left(=5\sqrt{5}\right)$	M1
	Vector $\overrightarrow{AB}=\begin{bmatrix}3--2\\12-2\end{bmatrix}\left(=5\begin{bmatrix}1\\2\end{bmatrix}\right)$	M1
	$\left(\left\lvert\overrightarrow{CX}\right\rvert=4\sqrt{5}\Rightarrow\right)\overrightarrow{CX}=4\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}4\\8\end{bmatrix}$	ddM1
	$\Rightarrow$ Coordinates of X are $(1,8)+\ (4,8)=(5,16)$	M1A1
	$(m=5\quad n=16)$	A1 [7]
ALT3	$\dfrac{1}{2}\begin{vmatrix}17 & m & 1 & 17\\0 & n & 8 & 0\end{vmatrix}$	B1
	$\dfrac{1}{2}\begin{vmatrix}\text{"17"} & m & \text{"1"} & \text{"17"}\\0 & n & \text{"8"} & 0\end{vmatrix}=80\Rightarrow\dfrac{1}{2}\left(17n+\text{"8"}m-(\text{"136"}+n)\right)=80\left(\Rightarrow 2n+m=37\right)$	M1
	$\text{"2"}=\dfrac{n-8}{m-1}\Rightarrow[n=2m+6]$ oe **or** $y-2=\text{"2"}(x+2)\Rightarrow[y=2x+6]$ oe **or** $y-12=\text{"2"}(x-3)\Rightarrow[y=2x+6]$ oe	M1
	$\dfrac{1}{2}\begin{vmatrix}\text{"17"} & m & \text{"1"} & \text{"17"}\\0 & \text{"}2m+6\text{"} & \text{"8"} & 0\end{vmatrix}$ or $\dfrac{1}{2}\left(17(\text{"}2m+6\text{"})+\text{"8"}m-(\text{"136"}+\text{"}2m+6\text{"})\right)=80$	ddM1
	or $2(\text{"}2m+6\text{"})+m=37\Rightarrow m=$	M1
	$m=5,\ n=16$	A1A1 [7]
		Total 16 marks

Part	Mark	Notes
(a)	B1	For $p = 1$ OR $q = 8$ Allow anything that clearly implies these values eg (1, 8) or 1, 8
	B1	For $p = 1$ AND $q = 8$ Allow anything that clearly implies these values eg (1, 8) or 1, 8
(b)	B1	For the gradient of $l = 2$
	B1ft	For the gradient of $k = -\frac{1}{"2"}$. Ft their calculation of the gradient of l.
	M1	For the correct unsimplified equation for k, using their p and q and any **changed** gradient. If the candidate uses $y = mx + c$, a fully correct rearrangement to find c must be shown (this can be implied by a correct c for their equation) and concluded with the equation of the line written.
	A1 cso	For the equation of k in the required form. Minimum steps shown, no errors or omissions. This must from a correct p and q
Watch for methods using vectors. If the candidate arrives at the correct unsimplified equation with no obviously incorrect work, following vector work, allow B1 B1ft M1 and the final A1 if the equation is given in the correct form. (See example in Practice)		
(c)	B1	For the x coordinate of $D = 17$
	M1	For a correct **method** to find the length of CD using their p and q and their 17
	A1	For the correct length of $\sqrt{320}$ oe
(d)	B1	Any correct unsimplified calculation for CX
	M1	For correctly using Pythagoras to form an equation in terms of m and n or x and y. Follow through their CX (ie coming from correct calculation with their p, q, their point D).
	M1	For an unsimplified equation in terms of m and n or x and y correctly using the gradient of l. Follow through their gradient for l, their p and their q. Allow working in p, q or x, y Note this work is sometimes being seen in (b). **As long as it is used in part (d),** the equation found in (b) may be given credit.
	ddM1	For eliminating either m or n from either equation and forming a 3TQ. Allow errors in processing if an initial correct method following from 2 correct unsimplified equations to eliminate one of the variables is seen. Allow working in p, q or x, y Dependent on both previous method marks. Note, the previous M mark is for the unsimplified equation, so if simplified incorrectly, this can be used here to gain this mark (general principle of marking).
	M1	For a minimally acceptable attempt to solve (see general guidance) their 3TQ This mark can be implied from a correct value of m = 5. If the quadratic is not the correct quadratic, method must be shown.
	A1	For one of $m = 5$ or $n = 16$
	A1	For both $n = 16$ and $m = 5$
ALT1	First B, final 2 A marks – as main scheme	
	M1	For correctly using Pythagoras to form an equation in terms of m and n or x and y. Follow through their CX and CD (ie coming from correct calculation with their p, q, their point D).
	M1	For an unsimplified equation in terms of m and n or x and y correctly using the gradient of l. Follow through their gradient for l, their p and their q. Allow working in p, q or x, y Note this work is sometimes being seen in (b). **As long as it is used in part (d),** the equation found in (b) may be given credit.
	ddM1	For eliminating either m or n from either equation and forming a 3TQ. Allow errors in processing if an initial correct method following from 2 correct unsimplified equations to eliminate one of the variables is seen. Allow working in p, q or x, y Dependent on both previous method marks. Note, the previous M mark is for the unsimplified equation, so if simplified incorrectly, this can be used here to gain this mark (general principle of marking).
	M1	For a minimally acceptable attempt to solve (see general guidance) their 3TQ This mark can be implied from a correct value of m = 5. If the quadratic is not the correct quadratic, method must be shown.

ALT2	B1	Any correct unsimplified calculation for *CX*
	M1	Correct method to find the length of *AB*
	M1	Correct method to find vector *AB*
	ddM1	Correctly deduces the relationship between their vectors *CX* and *AB* Dependent on both previous method marks.
	M1	Correct method to find the coordinates of *X* using their vector work.
	A1	For one of $m = 5$ or $n = 16$
	A1	For both $n = 16$ and $m = 5$
ALT3	B1	For a fully correct array as shown oe
	M1	For placing their array (though ft their *p* and *q* and their "17") = 80 and correctly multiplying out the discriminant to form an unsimplified equation in *m* and *n*
	M1	For an unsimplified equation in terms of *m* and *n* or *x* and *y* correctly using the gradient of *l*. Follow through their gradient for *l*, their *p* and their *q*. Allow working in *p*, *q* or *x*, *y* Note this work is sometimes being seen in (a). **As long as it is used in part (d),** the equation found in (a) may be given credit.
	ddM1	For eliminating either *m* or *n* seen either in the array or in their equation formed. Allow errors in processing if an initial correct method following from 2 correct unsimplified equations to eliminate one of the variables is seen. Allow working in *p, q* or *x, y* Dependent on both previous method marks. Note, the previous M mark is for the unsimplified equation, so if simplified incorrectly, this can be used here to gain this mark (general principle of marking).
	M1	For solving their linear equation, allow one error in processing, leading to a value of *m* or *n*
	A1	For one of $m = 5$ or $n = 16$
	A1	For both $n = 16$ and $m = 5$
If a partial method is seen, with correct answers not from obvious incorrect working, marks may be given (general point of marking unless precluded from the mark scheme for a question). If a partial method is seen and marks cannot be fitted to the main or ALT schemes – send to review.		

Question	Scheme	Marks
7(a)	$(a=)\dfrac{4^2}{4}-3\sqrt{4}+8=6$	B1 cso [1]
(b)	$\left(\dfrac{dy}{dx}=\right)\dfrac{2x}{4}-3\times\dfrac{1}{2}\times x^{-\frac{1}{2}}$ oe $\left(x=4\Rightarrow\dfrac{dy}{dx}=\right)\dfrac{2\times4}{4}-3\times\dfrac{1}{2}\times4^{-\frac{1}{2}}=\dfrac{5}{4}\Rightarrow$ $M_n=-\dfrac{1}{"\frac{5}{4}"}$ $y-6="-\dfrac{4}{5}"(x-4)\Rightarrow 5y+4x-46=0$ *	M1 dM1A1 M1 (now matches ePen) M1A1 cso [6]
(c)	Area under C $\left(A_C=\int_1^4\left(\dfrac{x^2}{4}-3\sqrt{x}+8\right)\ dx\right)$ $=\left[\dfrac{x^3}{3\times4}-\dfrac{3\times x^{\frac{3}{2}}}{\frac{3}{2}}+8x\right]_1^4$ $=\left[\dfrac{4^3}{3\times4}-\dfrac{3\times4^{\frac{3}{2}}}{\frac{3}{2}}+8\times4\right]-\left[\dfrac{1^3}{3\times4}-\dfrac{3\times1^{\frac{3}{2}}}{\frac{3}{2}}+8\times1\right]$ $=\dfrac{61}{4}$ Area under the line $5\times0+4x-46=0\Rightarrow x(=11.5)$ $A=\dfrac{1}{2}\times("11.5"-4)\times6=\dfrac{45}{2}$ **ALT** $\int_4^{11.5}\left(-\dfrac{4}{5}x+\dfrac{46}{5}\right)dx=\left[-\dfrac{4}{5\times2}x^2+\dfrac{46}{5}x\right]_4^{11.5}$ $=\left(-\dfrac{4}{5\times2}\times11.5^2+\dfrac{46}{5}\times11.5\right)-\left(-\dfrac{4}{5\times2}\times4^2+\dfrac{46}{5}\times4\right)$ Required area $=\left(\dfrac{61}{4}+\dfrac{45}{2}=\right)\dfrac{151}{4}(=37.75)$ oe	 M1 M1 A1 M1A1 A1 [6]

ALT	Line - curve	
	$\int_1^4\left(\left(-\frac{4}{5}x+\frac{46}{5}\right)-\left(\frac{x^2}{4}-3\sqrt{x}+8\right)\right)\mathrm{d}x$ or $\int_1^4\left(-\frac{x^2}{4}+3\sqrt{x}-\frac{4}{5}x+\frac{6}{5}\right)\mathrm{d}x$	
	$\left[-\frac{4}{5\times 2}x^2+\frac{46}{5}x-\frac{x^3}{4\times 3}+\frac{3\times x^{\frac{3}{2}}}{\frac{3}{2}}-8x\right]_1^4$ or $\left[-\frac{x^3}{4\times 3}+\frac{3\times x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{4}{5\times 2}x^2+\frac{6}{5}x\right]_1^4$	M1
	$\left(-\frac{4}{5\times 2}\times 4^2+\frac{46}{5}\times 4-\frac{4^3}{4\times 3}+\frac{3\times 4^{\frac{3}{2}}}{\frac{3}{2}}-8\times 4\right)-$ $\left(-\frac{4}{5\times 2}\times 1^2+\frac{46}{5}\times 1-\frac{1^3}{4\times 3}+\frac{3\times 1^{\frac{3}{2}}}{\frac{3}{2}}-8\times 1\right)$ or $\left(-\frac{4^3}{4\times 3}+\frac{3\times 4^{\frac{3}{2}}}{\frac{3}{2}}-\frac{4}{5\times 2}\times 4^2+\frac{6}{5}\times 4\right)-\left(-\frac{1^3}{4\times 3}+\frac{3\times 1^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{5\times 2}\times 4^2+\frac{6}{5}\times 1\right)$	M1
	$=\frac{127}{20}$	A1
	Area under the line	
	$5\times 0+4x-46=0\Rightarrow x(=11.5)$ $5\times y+4\times 1-46=0\Rightarrow x\left(=\frac{42}{5}\right)$	
	$A=\frac{1}{2}\times(\text{"}11.5\text{"}-1)\times\text{"}\frac{42}{5}\text{"}=\frac{45}{2}$ $A=\frac{1}{2}\times(\text{"}11.5\text{"}-1)\times\text{"}\frac{42}{5}\text{"}=\frac{441}{10}$	M1 A1
	ALT $\int_1^{\text{"}11.5\text{"}}\left(-\frac{4}{5}x+\frac{46}{5}\right)\mathrm{d}x=\left[-\frac{4}{5\times 2}x^2+\frac{46}{5}x\right]_1^{\text{"}11.5\text{"}}$	
	$=\left(-\frac{4}{5\times 2}\times\text{"}11.5\text{"}^2+\frac{46}{5}\times\text{"}11.5\text{"}\right)-\left(-\frac{4}{5\times 2}\times 1^2+\frac{46}{5}\times 1\right)=\frac{441}{10}$	
	$\left(\frac{441}{10}-\frac{127}{20}=\right)\frac{151}{4}$	A1
	Useful diagram 	
		Total 13 marks

Part	Mark	Notes
(a)	B1	Correct substitution, no errors and shows $a = 6$
(b)	M1	For an attempt to differentiate the given function. Simplification not required. See General Guidance for the definition of an attempt. No power of x to increase.
	dM1	For substituting the value of $x = 4$ into their derivative. Can be implied by sight of $\frac{5}{4}$ Dependent upon the previous method mark.
	A1	For the correct gradient of $\frac{5}{4}$
	M1	For finding the negative reciprocal of their gradient.
	M1	For forming the equation of the normal, using the given value for a and their **changed** gradient (ie not the gradient of the tangent). This is not a dependent mark, but the candidate may not use the gradient of the tangent and the gradient used must come from some differentiation work. If the candidate uses $y = mx + c$, a fully correct rearrangement to find c must be shown (this can be implied by a correct c for their equation) and concluded with the equation of the line written.
	A1 cso	For the correct equation of the line, minimum steps shown, no errors or omissions.
(c)	M1	For an attempt to integrate the given function. See General Guidance for the definition of an attempt. In this question at least one term correctly integrated and no power of t to decrease. Terms do not need to be simplified. Limits don't need to be present or correct.
	M1	For substituting the correct coordinates into their **changed** expression, the correct way around and subtracting. At least one correct substitution in one term of each limit fully shown. Their limits must be correct.
	A1	For the correct area under the curve. A correct answer here will imply M1 M1 A1 if the integration step has been shown and first M1 awarded. Solutions where the integration step has not been shown will score M0 M0 A0.
	M1	For a correct method to find the area of the triangle using their value for the intersection of line $L$with the x-axis . or Fully correct integration for line and substitution of limits of their 11.5 (following any attempt to find where L crosses the x-axis) and 4. May be implied by correct area. Terms do not need to be simplified.
	A1	For the correct area of the triangle
	A1	For the correct area of $\frac{151}{4}$

<table>
<tr><td rowspan="6">ALT</td><td>M1</td><td>For an attempt to integrate either of the functions as given – there must be a subtraction sign if listed separately. Also award this mark for integrating
$\int_1^4 \left(-\frac{x^2}{4}+3\sqrt{x}-\frac{4}{5}x+E\right)\mathrm{d}x \quad E \neq 0$ (Combines the constant terms incorrectly)
See General Guidance for the definition of an attempt. In this question at least one term correctly integrated and no power of t to decrease. Terms do not need to be simplified. Limits don't need to be present or correct.</td></tr>
<tr><td>M1</td><td>For substituting the correct coordinates into their changed expression, the correct way around and subtracting. At least one correct substitution in one term of each limit fully shown. Their limits must be correct.</td></tr>
<tr><td>A1</td><td>For $\frac{127}{20}$</td></tr>
<tr><td>M1</td><td>For a correct method to find the area of the large triangle using their value for the intersection of line $L$with the x-axis .
or
Fully correct integration for line and substitution of limits of their 11.5 (following any attempt to find where L crosses the x-axis) and 4. May be implied by correct area. Terms do not need to be simplified.</td></tr>
<tr><td>A1</td><td>For $\frac{441}{10}$</td></tr>
<tr><td>A1</td><td>For the correct area of $\frac{151}{4}$</td></tr>
<tr><td colspan="3">Where students have combined expression incorrectly as follows:
$\int_1^4 \left(\left(\frac{x^2}{4}-3\sqrt{x}+8\right)\pm\left(-\frac{4}{5}x+\frac{46}{5}\right)\right) \mathrm{d}x$ If the constant term is unsimplified, the highest mark will be up to M1 M1 A1 M0 A0 A0 if the work is seen for the first 3 marks as described above.

If they have simplified the constant term to give eg
$\pm\left(\frac{x^2}{4}-3\sqrt{x}+\frac{86}{5}\right)\pm\frac{4}{5}x$ or $\pm\left(\frac{x^2}{4}-3\sqrt{x}+\frac{86}{5}\right)\pm\frac{86}{5}x$
The highest mark will be M1 M0 A0 M0 A0 A0 if the first two terms in the curve and any constant term are integrated as per conditions above.</td></tr>
</table>

Question	Scheme	Marks
8 (a)	$a+ar=400$ or $\dfrac{a(1-r^2)}{1-r}=400$, $ar+ar^2=100$ oe Common methods 1) $r\left(a+ar\right)=100 \Rightarrow r\left(400\right)=100$ $r=\dfrac{1}{4}$ * 2) $a=\dfrac{400}{1+r} \Rightarrow \left(\dfrac{400}{1+r}\right)r+\left(\dfrac{400}{1+r}\right)r^2=100$ $\Rightarrow 400r+400r^2=100+100r \Rightarrow 4r^2+3r-1=0 \Rightarrow (4r-1)(r+1)\left(=0\right)$ $r=\dfrac{1}{4}$ * 3) $r=\dfrac{400-a}{a} \Rightarrow a\left(\dfrac{400-a}{a}\right)+a\left(\dfrac{400-a}{a}\right)^2=100 \Rightarrow$ $\Rightarrow 400-a+\dfrac{\left(400-a\right)^2}{a}=100 \Rightarrow$ $400-a^2+160000-800a+a^2=100a \Rightarrow a=320 \Rightarrow 320+320r=400$ $r=\dfrac{1}{4}$ *	B1B1 M1 A1cso M1 A1cso M1 A1cso [4]
ALT	$\left(\text{Let } G_1, G_2, G_3 \text{ be the first 3 terms}\right)$ $G_2+G_3=100$ $\left(G_1+G_2=400\right) \Rightarrow rG_1+rG_2=100$ $rG_1+rG_2=100 \Rightarrow \left(r\left(G_1+G_2\right)=100\right) \Rightarrow r(400)=100$ $r=\dfrac{1}{4}$ *	B1 B1 M1 A1*cso [4]
(b)	$\left(a=\right)\dfrac{300}{1-\left(\frac{1}{4}\right)^2}$ or $\dfrac{400}{1+\frac{1}{4}}$ or $\dfrac{100}{\frac{1}{4}+\left(\frac{1}{4}\right)^2}$ $=320$ *	M1A1 cso [2]
(c)	$S_\infty=\dfrac{320}{1-\frac{1}{4}}=\dfrac{1280}{3}$	M1A1 [2]
		Total 12 marks

Part	Mark	Notes
(a)	B1	For either equation shown correct *a* and *r* can be any letters throughout.
	B1	For both equations shown correct *a* and *r* can be any letters throughout.
	M1	For forming an equation eliminating *a* or *r* Allow one error in processing **such as a sign or arithmetical error, but not a 'cancellation'/simplification error**. Must be working with 2 correct equations. This mark can be awarded as soon as *a* or *r* are eliminated. Doesn't need simplification at this stage.
	A1 cso	For correctly solving and attaining $r = \frac{1}{4}$ minimum steps shown, no errors/omissions, ignore $r = -1$
ALT	B1	For either equation shown correct
	B1	For both equations shown correct
	M1	For multiplication of the first equation by *r* and formation of an equation in *r* Allow one error in processing. Must be working with 2 correct equations.
	A1	For $r = \frac{1}{4}$ minimum steps shown, no errors or omissions. Ignore work on any negative values
There are a number of different methods to do this, the four most commonly anticipated are shown. Mark to the following principles to gain the method mark: • One processing error only in any method **(M mark only, not A mark).** • Rearrange for *r* or *a* and correctly substitute into the other equation or such as method 1 to reach an equation in one variable only. • Rearrange the resulting equation so that an equation of the form ***br* = *c*** is reached. Note for the quadratic option, a factorisation will suffice. Note, if eliminating *r*, *a* must be found and the value of *a* then substituted into an appropriate equation. Methods where these principles can't be applied and thought worthy of credit **– send to review please**.		
(b)	M1	For using their expression for *a* with the correct *r*, to find a value for *a* Note, **for this question only**, work in (a) may be credited for this mark – **only if they eliminated *r* in their solution for part (a) and this is then used.**
	A1cso	For 320, no errors.
(c)	M1	For using the correct formula for the sum to infinity of a convergent series with the given values of *a* and *r* to find a value.
	A1	For the exact value of $\frac{1280}{3}$ oe or 426.67 or better (ie correctly rounded to more decimal places) or 426.6… (minimum 3 dots) or 426.66^r or $426.\dot{6}$
(d)	M1	Uses the correct formula for the sum of a geometric series, to set up **an inequality or equation,** allow < or > or = using the given values of *r* and *a*. Condone $\frac{1}{4}^n$
	dM1	For simplifying (allow errors in simplification) their **inequality or equation** in *n* to the form $\left(\frac{1}{4}\right)^n < d \quad d \neq 0$ or $4^n < d$ Allow < or > or =. Dependent on the 1st method mark. Condone poor bracketing with powers again.
	ddM1	For the correct use of logs and correct use of an inequality sign throughout, including the reversal at the appropriate point. Dependent on both previous method marks. This mark may not be awarded if 'd' is negative. If candidates give a final answer of (*n* =) 7 – this mark can be implied even if the inequality sign is not correctly reversed.
	A1	For (*n* =) 7 Note although *n* = 7 can imply ddM1 as described, it is unlikely to imply the first 2 marks as there must be some logs work (directed by the question).

(d)	$426.6 < \dfrac{320\left(1-\left(\frac{1}{4}\right)^n\right)}{1-\frac{1}{4}} \Rightarrow \left(\dfrac{1}{4}\right)^n < \dfrac{1}{6400}$ or $4^n < 6400$	M1dM1
	$\Rightarrow n > \dfrac{\log\left(\frac{1}{6400}\right)}{\log\left(\frac{1}{4}\right)}$ or $n > \log_{\frac{1}{4}}\left(\dfrac{1}{6400}\right)$ oe	ddM1
	$\Rightarrow n > 6.32... \Rightarrow n = 7$	A1 [4]
		Total 12 marks

Question	Scheme	Marks
9 (a)	$\left(\log_a 8=\frac{3}{4}\Rightarrow\right)a^{\frac{3}{4}}=8\left(\Rightarrow a=\left(\sqrt[3]{8}\right)^4\right)=16$	M1A1 [2]
(b)	$\left(3x\log_2 x-4\log_{16}8+6x\log_4 8-\log_2 x=\right)3x\log_2 x-\frac{4\log_2 8}{\log_2 16}+\frac{6x\log_2 8}{\log_2 4}-\log_2 x$ $\Rightarrow 3x\log_2 x-\log_2 8+3x\log_2 8-\log_2 x$ $=(3x-1)\log_2 x+(3x-1)\log_2 8$ or $3x\log_2 8x-(1)\log_2 8x$ $\Rightarrow(3x-1)\log_2 8x$ or $\log_2(8x)^{3x}+\log_2(8x)^{-1}=\log_2(8x)^{3x-1}$ *	M1 M1 M1A1 cso [4]
ALT	$\left(3x\log_2 x-4\log_{16}8+6x\log_4 8-\log_2 x=\right)3x\log_2 x-\frac{4\log_2 8}{\log_2 16}+\frac{6x\log_2 x}{\log_2 4}-\log_2 x$ $\Rightarrow 3x\log_2 x-\log_2 8+3x\log_2 8-\log_2 x=\log_2 x^{3x}-\log_2 8+\log_2 8^{3x}-\log_2 x$ $\left(\Rightarrow\log_2(8x)^{3x}-\log_2 8x\Rightarrow\right)\log_2\left(\frac{(8x)^{3x}}{8x}\right)$ or $\log_2\left((8x)^{3x}\times(8x)^{-1}\right)$ or $\log_2 x^{3x-1}8^{3x-1}=\log_2(8x)^{3x-1}$ *	M1 M1 M1A1 cso [4]

"Box 3" of part b
We will see unanticipated methods once live marking begins.
If the answer is correct and **there is no incorrect working**, check the work carefully, to ascertain if they've shown enough steps to demonstrate use of the three main log laws this question tests and award full marks. If in any doubt at all – the response **MUST** be sent to review.

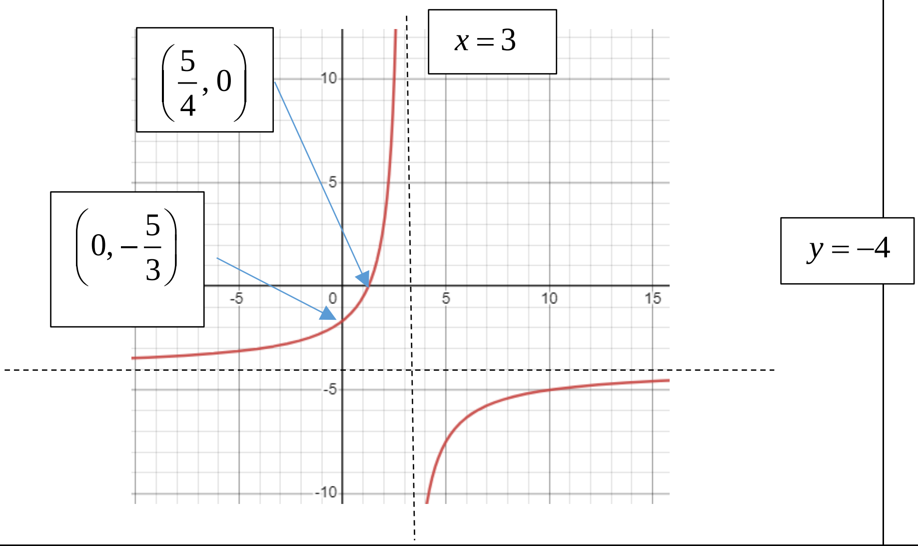
Other than this exception, please mark to the following rules.
Also use these rules if students don't gain the 2nd or 3rd M under the main or ALT schemes.

- M1 for any correct change of base to base 2
- M1 for any two correct applications of the power law or for $ax\log_2 8x+b\log_2 8x\Rightarrow\log_2(8x)^{ax+b}$ or $(ax+b)\log_2 8x\Rightarrow\log_2(8x)^{ax+b}$ $a,b\neq 0$
- M1 for any two correct applications of the addition or subtraction law

In each case – ignore any incorrect working.
Poor or incorrect bracketing may not be recovered in this question. (general principle of marking is usually that it can).

Question	Scheme	Marks
(c)	$\left[\log_2(8x)^{3x-1}=0\Rightarrow\log_2(8x)^{3x-1}=\log_2 8^0\right.$ or $\left.(3x-1)\log_2(8x)=0\right]$ $\Rightarrow 3x-1=0\Rightarrow x=\frac{1}{3}$ $\Rightarrow 8x=1\Rightarrow x=\frac{1}{8}$	M1A1 A1 [3]
		Total 9 marks

Part	Mark	Notes
(a)	M1	For undoing the log to obtain $a^{\frac{3}{4}}=8$ or $(a=)8^{\frac{4}{3}}$
	A1	For a = 16 M1 A1 may be awarded for just seeing a = 16 unless from a string of incorrect work. Poor notation may be seen.
(b)	M1	For correctly changing the base to base 2 on at least one log, anywhere in their work. This may be implied and the mark awarded for eg $4\log_{16}8 \Rightarrow \frac{4\log_2 8}{\log_2 16}$ or $\frac{4\log_2 8}{4}$ or $\log_2 8$ or $\frac{12}{4}$ or 3 or $6x\log_4 8 \Rightarrow \frac{6x\log_2 8}{\log_2 4}$ or $\frac{6x\log_2 8}{2}$ or $3x\log_2 8$ or $9x$
	M1	For $(3x-1)\log_2 x+(3x-1)\log_2 8$
	M1	For correctly applying the power log law to their expression of the form $(ax+b)\log_2 8x \quad a,b\neq 0$ In general, poor bracketing may be recovered, but as this question is a show question, it generally cannot be recovered eg do not accept $8x^{3x}$
	A1 cso	For the correct expression, **minimum steps as shown** with no errors or omissions.
ALT	M1	For correctly changing the base to base 2 on at least one log. This may be implied and the mark awarded for $4\log_{16}8 \Rightarrow \frac{4\log_2 8}{\log_2 16}$ or $\frac{4\log_2 8}{4}$ or $\log_2 8$ or $\frac{12}{4}$ or 3 or $6x\log_4 8 \Rightarrow \frac{6x\log_2 8}{\log_2 4}$ or $\frac{6x\log_2 8}{2}$ or $3x\log_2 8$ or $9x$
	M1	For $\log_2 x^{(3x)}-\log_2 8+\log_2 8^{(3x)}-\log_2 x$
	M1	For correctly applying the subtraction law to an expression of the form (see the MS for minimum steps) $\log_2 x^{(ax)}-\log_2 8^b+\log_2 8^{(ax)}-\log_2 x^b$ The step in brackets doesn't need to be shown. Do not permit only eg $\log_2\left(x^{3x}\div 8\times 8^{3x}\div x\right)$ as a sufficient minimum step. In general, poor bracketing may be recovered, but as this question is a show question, it generally cannot be recovered eg do not accept $8x^{3x}\times 8x^{-1}$
	A1	For the correct expression, **minimum steps as shown** with no errors or omissions. .
	SC4	For working on both sides **with no errors** and achieving lhs = rhs Note, this is an exception to what we normally allow in a show that question (ie working on both sides until agreement)
(c)	M1	For setting $3x-1=0$ or $8x=1$ This mark may be implied by a correct answer.
	A1	For either $x=\frac{1}{3}$ or $\frac{1}{8}$
	A1	For both $x=\frac{1}{3}$ and $\frac{1}{8}$

Question	Scheme	Marks
10(a)	$\left(ax-5=0,\ x=\frac{5}{4}\Rightarrow\right)a=4$ $\big($Line parallel to the y-axis is $x=3\Rightarrow\big)b=3$	B1 B1 {2]
(b)		B1 B1ftB1 B1ftB1ft [5]
(c)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{(3-x)(4)-(4x-5)(-1)}{(3-x)^2}=\left[\frac{7}{(3-x)^2}\right]$ (Gradient of l =) $\frac{7}{4}$ $\frac{7}{4}=\frac{7}{(3-x)^2}\Rightarrow(3-x)^2=4\Rightarrow 3-x=\pm2\Rightarrow x=1,5$ When x = 1, $y=-\frac{1}{2}$ and when x = 5, $y=-\frac{15}{2}$ Equation of line when x = 1: $y--\frac{1}{2}=\frac{7}{4}(x-1)\Rightarrow 4y-7x=-9$ Equation of line when x = 5: $y--\frac{15}{2}=\frac{7}{4}(x-5)\Rightarrow 4y-7x=-65$ $\Rightarrow -65<k<-9$	M1 B1 M1A1 M1 A1 (B1B1 on ePen) ddM1 ddM1 A1 [9]

ALT	$(y=)\dfrac{7x+k}{4}=\dfrac{"4"x-5}{"3"-x}$ oe $\quad 4\left(\dfrac{"4"x-5}{"3"-x}\right)-7x=k$ oe $16x-20=(7x+k)("3"-x)$ oe $16x-20=21x-7x^2+3k-kx \Rightarrow 7x^2+kx-5x-20-3k\left(=0\right)$ oe $\left(\Rightarrow 7x^2+(k-5)x-20-3k\left(=0\right)\right)$ oe $(k-5)^2-4(7)(-20-3k)\left(<0\right)$ $k^2+74k+585\left(<0\right)$ $(k=)\dfrac{-74\pm\sqrt{(74)^2-4\times(1)\times585}}{2}$ or $(k-9)(k-65)\left(=0\right)$ $-65<k<-9$	M1 A1(B1 on ePen) M1 A1 M1A1 (B1 B1 on ePen) ddM1 ddM1 A1 [9]

<table>
<tr><th>Part</th><th>Mark</th><th>Notes</th></tr>
<tr><td rowspan="2">(a)</td><td>B1</td><td>For the value of $a = 4$ or $b = 3$</td></tr>
<tr><td>B1</td><td>For the value of $a = 4$ and $b = 3$</td></tr>
<tr><td colspan="3">Note: These are independent of method marks, not given answers. Unless the answers come from very obviously incorrect working, these marks should be awarded for sight of the correct values.</td></tr>
<tr><td rowspan="7">(b)</td><td>B1</td><td>For a negative reciprocal curve drawn anywhere in the grid – there must be two branches present, they must not cross any asymptotes drawn and must not <u>obviously</u> ‘bend back’ on themselves. Mark intention.</td></tr>
<tr><td colspan="2">For the following marks, where candidates have used incorrect values of a and b, examiners will find using desmos.com or similar packages useful.
Type into desmos $y = (ax - b)/(b - x)$ and add ‘sliders’ for a and b</td></tr>
<tr><td>B1ft</td><td>For the horizontal asymptote of $y = -4$ drawn in the correct place and labelled with its equation or where the line passes through the axis.
There must be at least one branch of a reciprocal curve present which must not cross or obviously bend back from the asymptote(s) ignore any other curves present.
Allow follow through only of $y = -$ (their a) and only if other conditions met.</td></tr>
<tr><td>B1</td><td>For the vertical asymptote $x = 3$ drawn in the correct place.
There must be at least one branch of a reciprocal curve present which must not cross or obviously bend back from the asymptote(s), ignore any other curves present.</td></tr>
<tr><td>B1ft</td><td>A single curve (or even line) passing through $\left(0, -\frac{5}{3}\right)$ marked clearly on the graph as a coordinate or as a crossing point on the axis.
Ignore any other branches present.
Allow follow through only of $\left(0, -\frac{5}{\text{their}\, b}\right)$</td></tr>
<tr><td>B1ft</td><td>A single curve (or even line) passing through both $\left(\frac{5}{4}, 0\right)$ and $\left(0, -\frac{5}{3}\right)$ both marked clearly on the graph as a coordinate or as crossing points on the axes.
Ignore any other branches present.

Allow follow through only of $\left(0, -\frac{5}{\text{their}\, b}\right)$</td></tr>
</table>

(c)	M1	For applying the quotient (or product rule). • With two terms in the numerator subtracted either way around. • The differentiation on both terms must be correct. • The denominator must be squared. $\dfrac{(3-x)(4)-(4x-5)(-1)}{(3-x)^2}$ or $\dfrac{(4x-5)(-1)-(3-x)(4)}{(3-x)^2}$ (if using quotient rule) Or $4(3-x)^{-1}\pm(4x-5)(3-x)^{-2}$ (if using product rule)
	B1	For (Gradient of l =) $\dfrac{7}{4}$
	M1	For correctly setting their gradient of l = to their derivative and an attempt to solve to find 2 values of x. Allow one error in rearrangement. Their derivative must be of the form $\dfrac{rx+s}{(3-x)^2}$ s **or** r could be 0, but not both
	A1	For x = 1 **and** 5
	M1 (B1 on ePen)	For substitution of either of their x values into the equation of their curve.
	A1 (B1 on ePen)	For $y=-\dfrac{1}{2}$ **and** $y=-\dfrac{15}{2}$
	ddM1	For correctly forming an equation of the line for either pair of their values and their gradient and rearranging (allow errors in rearrangement) to the form $4y-7x=k$. Dependent on 1st 2 method marks.
	ddM1	For correctly forming an equation of the line for both pairs of their values and their gradient and rearranging (allow rearrangement errors) to the form $4y-7x=k$ Dependent on 1st 2 method marks.
	A1	For the correct inequality as shown.
ALT	M1	For rearrangement of l to the form $y=\dfrac{7}{4}x+c \quad c\neq 0$ and placing equal to their equation of C or substitution of the equation for curve C in the equation for l. Allow $4\left(\dfrac{"4"x-5}{"3"-x}\right)=k\pm 7x$ Condone using an inequality sign, follow through their a and b
	A1 (B1 on ePen)	For a correct unsimplified equation. **DO NOT** condone using an inequality sign
	M1	For an attempt to rearrange their equation of the form $mx+c=\dfrac{4x-5}{3-x} \quad m,c\neq 0$, to allow them to reach a “3TQ”, allow one error in rearrangement. Condone using an inequality sign By “3TQ”, we means terms in x, x^2 and constant, all on ‘one side’ even if the equals sign is missing or an incorrect inequality sign is shown, terms simplified where possible.
	A1	For the correct “3TQ” shown (oe), the term in x does not have to be factorised and = 0 may be omitted. “3TQ” defined as above
	M1 (B1 on ePen)	For the correct formulation of the discriminant from their “3TQ”. Inequality sign doesn’t need to be shown or can be incorrect.
	A1 (B1 on ePen)	For the correct unsimplified discriminant. Inequality sign doesn’t need to be shown or can be correct.
	ddM1	For a correct method to solve their quadratic equation in k, leading to a value of k. Dependent on first 2 method marks. Mark awarded for the correct **method.**
	ddM1	For a correct method to solve their quadratic equation in k, leading to two values of k. Dependent on 1st 2 method marks. Mark awarded for the correct **method.**
	A1	For the inequality shown.

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